Intuitionistic Description Logic and Legal Reasoning

Edward Hermann Haeusler
DI PUC-Rio

Valeria de Paiva
C.S. Univ. of Birmingham

Alexandre Rademaker
CMA FGV-Rio

***Abstract*—Classical Logic has been used as a basis for knowledge representation and reasoning in many domains. Legal Knowledge Representation is interesting due to the natural occurrences of conflicts among law systems, individual laws and cases, usually taken as logical inconsistencies. Due to its inherently normative feature, coherence (consistency) in legal ontologies is more subtle than in other domains. An adequate intuitionistic semantics for negation in a legal domain comes to the fore when we consider laws as individuals instead of propositions. This paper presents a version of Intuitionistic Description Logic designed for legal knowledge representation. The paper discusses a logical coherence analysis of "Conflict of Laws in Space", using our logic, and briefly compares this analysis to other logical approaches to Legal Reasoning.**

I. Introduction

This article proposes an alternative logical basis for Legal Reasoning (LR) in the context of AI. There are many aspects to be considered whenever we pose such a foundational question as what constitutes the logical basis for representing LR. It seems clear that the way LR is conceived is strongly related to the way laws (or "The Law") are represented. LR is strongly interconnected to Legal Knowledge Representation, i.e, to the Legal Ontology chosen, in a wider sense of the term *ontology*. What we see is that LR must have *ontological commitments*; it cannot be simply based on a "legal" logic. It must also make a commitment as how do we get to know what we are reasoning about. Our main goal here is to show how an intuitionistic version of the Description Logic ALC [1] can be used to handle adequately certain incoherent situations in LR.

In [2], Knowledge Representation KR is taken as a kind of *surrogate*, a substitute for the represented knowledge itself. According to that seminal work some fundamental roles that KR plays in AI are: (1) It is a set of *ontological commitments*; (2) It is a fragmentary theory of intelligent reasoning; (3) It is a medium for *efficient computation*; and finally (4) It is a medium of *human expression*. Applying these principles we conclude that:

1) LR should be based on logic. We focus on *reasoning* rather than on learning or data-mining. We want to generate human readable explanations for the reason why a statement is a valid legal statement about a given legal system.
2) The ontological commitments of LR should be guided by the underlying jurisprudence theory and judicial practice. But fundamentally, these ontological commitments need to pay attention to computational efficiency and feasibility.

When dealing with KR, questions about the adequacy of the *surrogate* arise. The surrogate must substitute the "real thing" as faithfully as possible. Considering principle 1, there cannot be inconsistencies in the set of sentences describing our knowledge. An inconsistent theory has no model, and hence, describes/represents nothing. Inconsistency is usually associated with formal versions of negation. The concept of logical negation is also essential when representing legal knowledge. Consistency seems to be more difficult to maintain when more than one coherent *law system* can judge a case. This is also called a *conflict of laws*. There are some traditional legal mechanisms to solve these conflicts; some of them state privileged fori, others consider jurisdiction, or law hierarchy or precedence between laws. Even using these legal mechanisms, coherence is still a major concern. As consistency is strongly related to logical negation, negation's role in LR is more subtle than it is in other domains. In this article we mainly investigate some inconsistencies that arise from the use of the excluded middle in Classical ALC, a core description logic. Description Logics (DLs) have been widely used in KR. DLs are basically conceptual languages. A DL knowledge base is defined on top of a set of concept terms plus a set of binary-relations, some statements about individuals (the $ABOX$), and a set of subsumption assertions between concepts (the $TBOX$). Entity Relationship models, UML specifications, and many other frameworks in Computer Science can be adequately represented by DL theories. Most of the DLs used are decidable. Althouth the computational complexity of DL satisfiability is at least PSPACE, there are good commercial reasoners to test satisfaction (truth) of a DL formula, and they can be used to test logical consequence of subsumptions with respect to a theory. Following principle 2, we want to use the expertise from the DL community to improve LR.

In section II, we show that our underlying jurisprudence theory is better served by the use of an intuitionistic version of ALC. The tailoring of this version, which we write as iALC for intuitionistic ALC, is also a contribution of this work. We show how iALC can be adequately used in LR.

Explained to a modal logician, ALC is formally equivalent to multimodal K [3]. Using the modern modal logic perspective, ALC with ABOXes can be taken as a hybrid logic. The assertion $pl\colon BR$, expressing the fact that pl is a nominal denoting an individual (law) inhabiting the extension of the concept BR, is modally taken as "the proposition BR holds at world pl". Keeping both readings of ALC in mind (as multimodal and as a hybrid system) helps to understand our definition of iALC and its use in LR. In this article, each individual law will be taken as a possible world (in a Kripke structure) and the collection of all individual laws (valid legal statements) forms our legal world. We choose "Conflict of Laws in Space", [1] to show how intuitionistic negation (in iALC) can help us deal with incoherent laws.

As discussed in [4], there are several possible ways of defining *constructive* description logics. Here we use a constructive version of ALC, based on the framework for constructive modal logics developed by Simpson in [5]. Our system is also similar to the constructive framework developed in [6] for Hybrid Logics, its immediate inspiration. A shorter version of this work was presented in [7].

II. Jurisprudence and Intuitionism

To know what should be the basic unit of law is a fundamental open question in jurisprudence theory. Any approach to law classification requires first answering this question ([8]). There are two main approaches to the question: (1) one can take all normative statements, as a whole, as "the law", or (2) one can take any legally valid statement as an *individual law*. The approach (1) carries the hard task of ruling a perfect world. The approach (2) seems to be nowadays predominant in legal philosophy and jurisprudence and owes its significance to the Legal Positivism tradition initiated by Hans Kelsen ([9]). The coherence of laws plays a central role in both. The main debate on whether coherence is built-in by the restrictions induced by Nature in an evolutionary way, or whether coherence should be the object of knowledge management, seems to favor the latter. The approach (1), in essence, is harder to be shared with jurisprudence principles, since it is mainly concerned in morally justify the law. The approach (2) seems to be more suitable to Legal AI. In fact, some Knowledge Engineering groups pursue the latter as a basis for defining legal ontologies. We follow this route. In what follows we shall use "valid legal statement" (VLS) to denote an individual law holding in our universe of discourse. Following Kelsen's jurisprudence, our universe of discourse is only inhabited by VLSs. For instance, if "Mary is liable" is a VLS it has to be an element of our legal world. Concepts denote particular legal systems or situations (collections of individual laws). For example, we can use BR to denode the set of VLS holding in Brazil. In particular, we may have $ml\colon BR$ meaning "Mary is liable" is a VLS in the set of Brazilian individual laws (BR).

In the sequel we discuss the role of the negation in Legal Knowledge Representation. We will take valid legal statements as individuals of a legal universe, instead of taking them as (deontic) propositions. We will compare classical negation to intuitionistic negation. The natural precedence between valid legal statements is the pre-order relationship that characterizes the Kripke Semantics for Intuitionistic Logic. In order to find out what happens in (classical) ALC, we check the simple task of negating an ALC concept. First recall that in classical ALC the following formula is valid, for any nominal i and concept C

$$i\colon C \sqcup i\colon \neg C$$

Consider Peter, a young man, who is not yet 18 years old. If pl is "Peter is liable" then $pl\colon BR$ cannot be the case, since the legal age in Brazil is 18. From the negation of $pl\colon BR$ and

$$pl\colon BR \sqcup pl\colon \neg BR$$

we have $pl\colon \neg BR$. Since in classical ALC the negation of a concept is interpreted as its set theoretical complement, this conclusion is too strong, it says that "Peter is liable" is a VLS that holds outside Brazil. For, if BR is interpreted as the set of VLS holding in Brazil then $\neg BR$ is the complement of this set, that is the set of VLS that holds outside Brazil. But $pl\colon \neg BR$ is not always the case, there might be no Peter at all, outside Brazil. Thus, as is well-known, negation in classical ALC presents problems for LR. Now considering an intuitionistic version of ALC, we do not have that $pl\colon \neg BR$ is a logical consequence of the above formalization. The semantics of negation in intuitionistic logic uses possible worlds, as the one for modal logic, and an hereditary relationship between worlds. In terms of intuitionistic semantics, $pl\colon \neg BR$ holds whenever every VLS accessible [2] from pl does not belong to BR. The relationship between VLSs is the natural precedence existing between laws. For example, "Peter is president of a company" must be preceded by pl, no one can be president of a company without being legally liable. Thus, the proposition $pl\colon \neg BR$, only takes into account VLSs that succeed pl in order to find out whether "Peter is liable" does belong to $\neg BR$. Our conclusion is that intuitionistic negation deals with classification of concepts better than classical negation whenever one has a natural precedence between VLS. The term "natural precedence" comes from the fact that it is a precedence related to the logic, not a precedence related to other arbitrary modeling considerations.

We can say that our logical approach to LR has a strong bias to static analyses of the legal world. Cases that are not yet sentenced cannot be taken into account. However, VLSs related to the case, as proofs that are legally part of a trial, can be considered. Thus, our approach follows principle 2 above.

III. The system iALC

Our system is based on the framework for intuitionistic modal logics proposed by Simpson [5] and called IML (intuitionistic modal logic). These modal logics arise from interpreting the usual possible worlds definitions in an intuitionistic meta-theory. The main benefit of these Natural Deduction systems when compared to traditional axiomatizations is their susceptibility to proof-theoretic techniques. Strong normalization and confluence results are proved for all of the systems

[1]This legal term is used to mean that the laws of different countries (or different jurisdictions), on the subject-matter to be decided, are in opposition to each other; or that certain laws of the same country are contradictory.

[2]A world a is accessible from world b if a is related by means of the hereditary relationship to b.

described by Simpson. On the downside the basic structure of Natural Deduction needs to be extended to deal with assumptions of the form *the world x is R-related to world y*, which is written as a second kind of formula xRy.

We also use ideas from [6], where the framework IHL, for *intuitionistic hybrid logics* is introduced. Hybrid logics add to usual modal logics a new kind of propositional symbol, the nominals, and also the so-called satisfaction operators. A nominal is assumed to be true at exactly one world, so a nominal can be considered the name of a world. If x is a nominal and X is an arbitrary formula, then a new formula $x\colon X$ called a satisfaction statement can be formed. The part $x\colon$ of $x\colon X$ is called a satisfaction operator. The satisfaction statement $x\colon X$ expresses the fact that the formula X is true at one particular world, namely the world at which the nominal x is true. Out of these tightly connected systems of intuitionistic modal IML and hybrid logics IHL, we want to carve out our logic. iALC concepts are described as:

$$
\begin{array}{lll}
C, D ::= & A \mid \bot \mid \top \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid C \sqsubseteq D \\
 & \mid \exists R.C \mid \forall R.C \\
F ::= & C \sqsubseteq D \mid a\colon C \mid aRb
\end{array}
$$

where A stands for an atomic concept, R for an atomic role, F for a formula and a, b for individuals (in hybrid logic reading, nominals). We could have used distinct symbols for subsumption of concepts and the subsumption concept constructor but this would blow-up the calculus presentation.

A constructive interpretation of iALC is a structure $\mathcal{I}$ consisting of a non-empty set $\Delta^{\mathcal{I}}$ of entities in which each entity represents a partially defined individual; a refinement pre-ordering $\preceq^{\mathcal{I}}$ on $\Delta^{\mathcal{I}}$, i.e., a reflexive and transitive relation; and an interpretation function $\cdot^{\mathcal{I}}$ mapping each role name R to a binary relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ and each atomic concept A to a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ which is closed under refinement, i.e., $x \in A^{\mathcal{I}}$ and $x \preceq^{\mathcal{I}} y$ implies $y \in A^{\mathcal{I}}$. The interpretation $\mathcal{I}$ is lifted from atomic $\bot, A$ to arbitrary concepts via:

$$
\begin{array}{rl}
\top^{\mathcal{I}} & =_{df} \Delta^{\mathcal{I}} \\
(\neg C)^{\mathcal{I}} & =_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. x \preceq y \Rightarrow y \notin C^{\mathcal{I}}\} \\
(C \sqcap D)^{\mathcal{I}} & =_{df} C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} & =_{df} C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(C \sqsubseteq D)^{\mathcal{I}} & =_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. (x \preceq y \text{ and } y \in C^{\mathcal{I}}) \Rightarrow y \in D^{\mathcal{I}}\} \\
(\exists R.C)^{\mathcal{I}} & =_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. x \preceq y \\
 & \quad \Rightarrow \exists z \in \Delta^{\mathcal{I}}. (y, z) \in R^{\mathcal{I}} \text{ and } z \in C^{\mathcal{I}}\} \\
(\forall R.C)^{\mathcal{I}} & =_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. x \preceq y \\
 & \quad \Rightarrow \forall z \in \Delta^{\mathcal{I}}. (y, z) \in R^{\mathcal{I}} \Rightarrow z \in C^{\mathcal{I}}\}
\end{array}
$$

Our setting simplifies [10], since iALC satisfies (like classical ALC) $\exists R.\bot = \bot$ and $\exists R.(C \sqcup D) = \exists R.C \sqcup \exists R.D$. We have no use for nested subsumptions.

Simpson's system captures in the rules the intuitions of the modalities over possible worlds. We adapt it to the description logic and the rules are in fig. 1. There are two main modifications from usual, non-labelled sequent calculus: [3] we add the labels, which intuitively describe the world where the formula (in our case the concept) is being asserted; and we use a new kind of premise in our deductive system, fig. 1, assertions of the form xRy, which mean that the role R relates worlds x and y. The main modification comes for the modal rules, which are now role quantification rules. We must keep the intuitionistic constraints for modal operators. Rules $\forall$-right and $\exists$-left have the usual condition that y is not in the conclusion.

[3] The reader may want to read Proof Theory books, for example, Takeuti, Jean-Yves Girard, Samuel Buss and Jan von Plato.

In [11] we provide a complete and sound Hilbert-style axiomatization for the iALC system in Fig. 1.

Theorem 1: The sequent calculus described in Fig. 1 is sound and complete for TBox reasoning, that is $\Theta, \emptyset \models C$ if and only if $\Theta \Rightarrow C$ is derivable in our axiom system.

Assertions like $x\colon C$ and xRy are important for LR in our modeling, as we show in the next section. The complexity analysis of satisfiability in iALC has to consider this kind of assertions too. We prove that iALC is PSPACE complete by adapting the game defined in [12] to our system. The game in [12] is a 2-person game of polynomial size on the size of the proposed sequent. One of the players has a winning strategy if and only if the proposed sequent is satisfiable. Determining existence of winning strategies in polynomial sized 2-person games can be implemented by PTIME Alternating Turing Machines, and the later are implemented by PSPACE ordinary Turing machines. This provides our upper bound. The lower bound is provided by the well-known theorem of Ladner on PSPACE completeness of Intuitionistic Logic and the logics between K and S4 and their fusions. Finally, as PSPACE=co-PSPACE, we conclude that provability of iALC sequents is also PSPACE-complete.

Theorem 2: The system iALC is decidable as far as provability and satisfiability are concerned. These problems can be shown PSPACE-complete.

IV. Private International Law

A concept symbol C, in a description logic language, is associated to a subset of *legal statements* representing a *kind of* legal situation. Roles in the description logic language are associated to relations between these *legal situations*, imposed by the relationship between pairs of valid *legal statements*.

We provide some details of how we model the legal situation known as "Conflict of Laws in Space" within the scope of *Private International Law*.

Consider the following situation: *Peter and Maria signed a renting contract. The subject of the contract is an apartment in Rio de Janeiro. The contract states that any dispute will go to court in Rio de Janeiro. Peter is 17 and Maria is 20 years old. Peter lives in Edinburgh and Maria lives in Rio.*

In order to proceed with our formal reasoning, the legal statement (1) *Maria and Peter have contractual obligations and rights to each other concerning an apartment in Rio de Janeiro* has to be valid. VLSs are the only individuals. There is no invalid *legal statement*. This follows from the foundations of jurisprudence assumed in this article. We will denote by cmp the legal statement (1) above. As before, let us denote by BR the set of (valid) individual legal statements in Brazil, and, by SC the set corresponding to $Scotland$. Since 18 years old is the legal age in Brazil, there is no individual legal statement about Peter in Brazil. Thus, the

$$\frac{}{\Gamma, x\colon \alpha \Rightarrow x\colon \alpha} \qquad \frac{}{\Gamma, x\colon \bot \Rightarrow \delta} \qquad \frac{}{xRy, \Gamma \Rightarrow xRy} \qquad \frac{\Gamma, x\colon \forall R.\alpha, y\colon \alpha, xRy \Rightarrow \delta}{\Gamma, x\colon \forall R.\alpha, xRy \Rightarrow \delta}\ \forall\text{-l}$$

$$\frac{\Gamma, xRy \Rightarrow y\colon \alpha}{\Gamma \Rightarrow x\colon \forall R.\alpha}\ \forall\text{-r} \qquad \frac{\Gamma, xRy, y\colon \alpha \Rightarrow \delta}{\Gamma, x\colon \exists R.\alpha \Rightarrow \delta}\ \exists\text{-l} \qquad \frac{\Gamma \Rightarrow xRy \qquad \Gamma \Rightarrow y\colon \alpha}{\Gamma \Rightarrow x\colon \exists R.\alpha}\ \exists\text{-r}$$

$$\frac{\Delta_1 \Rightarrow \alpha \qquad \Delta_2, \beta \Rightarrow \gamma}{\Delta_1, \Delta_2, \alpha \sqsubseteq \beta \Rightarrow \gamma}\ \sqsubseteq\text{-l} \qquad \frac{\Delta, \alpha \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqsubseteq \beta}\ \sqsubseteq\text{-r} \qquad \frac{\Delta, \alpha, \beta \Rightarrow \gamma}{\Delta, \alpha \sqcap \beta \Rightarrow \gamma}\ \sqcap\text{-l} \qquad \frac{\Delta \Rightarrow \alpha \qquad \Delta \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqcap \beta}\ \sqcap\text{-r}$$

$$\frac{\Delta, \alpha \Rightarrow \gamma \qquad \Delta, \beta \Rightarrow \gamma}{\Delta, \alpha \sqcup \beta \Rightarrow \gamma}\ \sqcup\text{-l} \qquad \frac{\Delta \Rightarrow \alpha}{\Delta \Rightarrow \alpha \sqcup \beta}\ \sqcup_1\text{-r} \qquad \frac{\delta \Rightarrow \gamma}{\exists R.\delta \Rightarrow \exists R.\gamma}\ \text{p-}\exists \qquad \frac{\Delta \Rightarrow \gamma}{\forall R.\Delta \Rightarrow \forall R.\gamma}\ \text{p-}\forall$$

Fig. 1. The System iALC: logical rules ($\sqcup_2$-r analogous)

statement *Maria is of legal age*, mla for short, is in BR, and pla (*Peter is of legal age*) is in SC. There is a natural precedence relation between legal statements, only legally capable individuals have civil obligations. In other words, $pla \preceq cmp$ and $mla \preceq cmp$. Let PIL_{BR} be the set of legal statements in Brazil describing its *Private International Law*. Of course we have $PIL_{BR} \sqsubseteq BR$. By its very nature, PIL_{BR} is a disjunction of concepts of legal statements that subsumes $\exists LexDomicilium.Abroad$. It is worth noticing that *Private International Law* (PIL) relates legal statements in different contexts, locations, time, etc. Thus each member of PIL corresponds to a specific context, here a geographical living place. $Abroad$ is the union of the legal statements holding in each country, but Brazil. *LexDomicilium* is a legal connection, a relationship between laws in jurisprudence terminology, written $LexD$ in the proof. The pair of legal statements $\langle pla, pla\rangle$ is in *LexDomicilium*, since Peter lives in Scotland, abroad as far as Brazil is concerned. Summing up, we have the following set of axioms: $\Delta = \{mla\colon BR, pla\colon SC, pla \preceq cmp, mla \preceq cmp, pla\ LexDom\ pla\}$ and $\Omega = \{PIL_{BR} \Rightarrow BR, SC \Rightarrow Abroad, \exists LexDom.Abroad \sqcup \ldots \Rightarrow PIL_{BR}, mla\colon BR, pla\colon BR \Rightarrow cmp\colon BR\}$.[4]

Ω displays the axioms stated in sequent form, whilst Δ is the set of concepts that we use to prove that $cmp\colon BR$. We start with the derivation presented in Fig. 2, where the sequents in Ω are used freely. The name of the concepts were shortened due to space limitations for displaying the proof tree.

We already know that $\Delta \Rightarrow mla\colon BR$, since $mla\colon BR$ occurs in Δ. Naming as Π the proof of Fig. 2, and, using the axioms from Δ, we conclude that $cmp\colon BR$, Fig. 3.

We conclude that $contract \in BR$, for each legal statement generalizing $contract$, with regard to $\preceq$, namely pla and mla, is in BR. For the interesting case we note that $pla\colon \exists LexDomicilium.SC \sqsubseteq PIL_{BR} \sqsubseteq BR$, by the definition of $\exists R.C$ concepts. This argumentation can be obtained from the proof. In fact it reflects each step in the proof.

If one uses ALC instead of iALC in the example formalization, one needs to consider a legal ontology involving non-valid *Legal Statements*, and hence an *ad hoc* ontology regarding jurisprudence main concepts. Dealing with non-valid legal statements will also increase the complexity of the ontology considered. Of course we simplified our example, since it only considers pla and mla as succeeding $contract$. In a real ontology, many more statements would have to be considered, for example $Maria\text{-}owns\text{-}the\text{-}apartment$ among them. This, we believe, would turn much more complex the classic ALC case.

The sequent calculus for iALC is inspired by [13] where labels are used for context controlling. This mechanism is shown ([14]) to be useful when defining a Natural Deduction system for ALC. For iALC we have already labels for the individuals. So here, we present the version without labels for roles.

V. Related Work

This work is an *alternative* logical basis for LR. In this section we briefly discuss two other main approaches: (1) The Deontic Logic approach, and, (2) The Defeasible Logic approach.

Many approaches to LR using deontic modalities have been proposed, from von Wright's seminal paper [15] to Mally, who was the first to envisage a logic for norms. While the motivation is clear, deontic reasoning seems to have almost the same problems as their non-deontic counter-parts. The many paradoxes that are known from philosophical logic, concerning moral and/or legal questions, are not solved with deontic logic. In fact, they are only concisely expressed in deontic logic. Alchourrón suggests that a norm should not always have assigned truth values. We agree with this, but our approach is distinct, since for us a norm never has a truth value. In this way we are closer to Kelsen's dichotomy (valid law vs. invalid law) than to Alchourrón. It is important to note that the pure use of a deontic logic has been shown to be inadequate to accomplish the task of formalizing legal reasoning. Deontic logic does not seem properly to distinguish between the normative status of a situation and the normative status of a norm (rule).

In Sartor's work [16], each law is defined as a defeasible rule (or sentence). A law has the general form $\alpha \leftarrow \beta_1 \wedge \ldots \beta_n \wedge S(\neg\gamma_1) \ldots \wedge S(\neg\gamma_k)$, where α is the conclusion, each β_i is a *principal fact*, and each γ_j is a *secondary fact*. In a trial, the onus of the proof of a principal fact is on the *plaintiff*,

[4] The axiom $mla\colon BR, pla\colon BR \Rightarrow cmp\colon BR$ states that the contract is valid in Brazil whenever both agents are liable in Brazil. In fact, this is an instance of a general law principle.

$$\dfrac{\dfrac{\dfrac{\exists LexD.Abroad \Rightarrow \exists LexD.Abroad}{\exists LexD.Abroad \Rightarrow PIL_{BR}}\,\sqcup\text{-r+cut} \qquad PIL_{BR} \Rightarrow BR}{\exists LexD.Abroad \Rightarrow BR}\,cut \qquad \dfrac{\dfrac{\Delta \Rightarrow pla\colon SC}{\Delta \Rightarrow pla\colon Abroad}\,\sqcup\text{-R} \qquad \Delta \Rightarrow pla\ LexD\ pla}{\Delta \Rightarrow pla\colon \exists LexD.Abroad}\,\exists\text{-R}}{\Delta \Rightarrow pla\colon BR}\,indi\text{-R}$$

Fig. 2. Derivation of $\Delta \Rightarrow pla\colon BR$

$$\dfrac{\dfrac{\dfrac{\Pi}{\Delta \Rightarrow pla\colon BR} \qquad mla\colon BR, pla\colon BR \Rightarrow cmp\colon BR}{\Delta, mla\colon BR \Rightarrow pla\colon BR}\,cut \qquad \Delta \Rightarrow mla\colon BR}{\Delta \Rightarrow cmp\colon BR}\,cut$$

Fig. 3. Derivation of $\Delta \Rightarrow cmp\colon BR$

while the onus of the proof of any of the secondary facts γ_j, is on the *defendant*. The law can be applied, if and only if, the defendant cannot provide proofs for the secondary facts. This approach is clearly dynamic, it aims at showing how a law can be used in a trial. Each law corresponds to a defeasible rule at the logical level. There are other approaches to LR using defeasible reasoning that are similar to [16]. Comparing these approaches to ours, we say that ours is static, we can only represent the legal world by means of the laws. In our setting, reasoning is conducted as if the legal proofs admitted at the trial were already laws (individually valid legal statements). We are able to express, as [16], the reason why the judge provided the sentence, but we cannot provide any clues about the dynamics of the trial. On the other hand, since we do not use defeasible reasoning, our approach is logically simpler.

Defeasible logic has linear time algorithms for derivability, however, the language is restrict, there is only negation on atoms, the implications have atomic conclusions and there is no disjunction. For this language, there are also linear time algorithms for ALC. But ALC is not a defeasible logic. Anyway, it is worth noting that no linear time defeasible logic exists for dealing with the full propositional language since classical propositional logic is NP-complete. Our calculus takes care of cyclic TBoxes, since the axioms of a given theory will appear as additional (non-logical) top-sequents in a proof.

VI. Conclusions

We used iALC to provide an alternative, and, we claim, more appropriate, definition of subsumption in the legal reasoning domain. The system iALC deals with the jurisprudence approach which consists of considering all (possible) legally valid individuals statements as laws. Conflict of laws is formalized by means of iALC, by showing its adequacy to perform coherence analysis in legal AI. The sequent calculus helps in this task. Our approach is different from [17], where the reasoning is by means of argumentation and opposition of propositions. Our basis is quite distinct, since we do not consider "laws" as propositions. Our approach is static while Sartor's approach is dynamic, but this also means that our logical basis is simpler, and consequently, computational support should be easier.

Our calculus first appeared in [11]. The example shown here was roughly modeled by the authors in [18]. Its use in formalizing the situation of conflict of laws in space is the novelty of this paper. In future work we intend to compare our results to the ones of [19], [20], [21] and [22].

References

[1] F. Baader, D. Calvanese, D. McGuinness, D. Nardi, and P. Patel-Schneider, Eds., *The Description Logic Handbook*. Cambridge Univ. Press, 2003.

[2] R. Davis, H. Shrobe, and P. Szolovits, "What is a knowledge representation?" *AI Magazine*, vol. 14, no. 1, p. 17, 1993.

[3] K. Schild, "A correspondence theory for terminological logics: preliminary report," in *Proc. of the 12th IJCAI, IJCAI'91*. San Francisco, CA, USA: Morgan Kaufmann Publishers Inc., 1991, pp. 466–471.

[4] V. de Paiva, "Constructive description logics: what, why and how," Xerox Parc, Tech. Rep., 2003.

[5] A. Simpson, "The proof theory and semantics of intuitionistic modal logic," Ph.D. dissertation, University of Edinburgh, December 1993.

[6] T. Braüner and V. de Paiva, "Intuitionistic hybrid logic," *JAL*, vol. 4, no. 3, pp. 231–255, 2006.

[7] E. H. Haeusler, V. de Paiva, and A. Rademaker, "Intuitionistic logic and legal ontologies," in *Proc. JURIX 2010*. IOS Press, 2010, pp. 155–158.

[8] J. Raz, "Legal principles and the limits of law," *YLJ*, vol. 81, pp. 823–854, 1972.

[9] H. Kelsen, *General theory of norms*. USA: Oxford Univ. Press, 1991.

[10] M. Mendler and S. Scheele, "Towards constructive DL for abstraction and refinement," *JAR*, vol. 44, no. 3, pp. 207–243, 2010, proc. 21st Intl DL Workshop.

[11] V. de Paiva, E. H. Hausler, and A. Rademaker, "Constructive description logic: Hybrid-style," in *Proc. HyLo'2010*, 2010.

[12] C. Areces, P. Blackburn, and M. Marx, "A road-map on complexity for hybrid logics," in *LNCS, 1683*, 1999, pp. 307–321.

[13] A. Rademaker, E. H. Haeusler, and L. C. Pereira, "On the proof theory of $\mathcal{ALC}$," in *The Many Sides of Logic. Proc of 15th EBL*. London: College Publications, 2008, vol. 21, pp. 273–285.

[14] A. Rademaker and E. Haeusler, "Providing a proof-theoretical basis for explanation: A case study on uml and alcqi reasoning," *JUCS*, vol. 16, pp. 3016–3042, 2010.

[15] G. von Wright, "Deontic logic," *Mind*, vol. 60, no. 237, pp. 1–15, 1951.

[16] G. Sartor, "The structure of norm conditions and nonmonotonic reasoning in law," in *Proc. 3rd ICAIL*, ACM. ACM Press, 1991, pp. 155–164.

[17] P. M. Dung and G. Sartor, "A Logical Model of Private International Law," *Deontic Logic in CS, LNAI 6181*, pp. 229–246, 2010.

[18] E. H. Haeusler, V. de Paiva, and A. Rademaker, "Using intuitionistic logic as a basis for legal ontologies," in *Proc. of the 4th LOAIT Workshop*. Fiesole, Florence, Italy: European Univ. Institute, 2010, pp. 69–76.

[19] C. E. Alchourron and A. A. Martino, "Logic without truth," *Ratio Juris*, vol. 3, no. 1, pp. 46–47, Jun 1990.

[20] J. Carmo and A. Jones, "Deontic logic and contrary-to-duties," *Handbook of Philosophical Logic*, vol. 8, pp. 265–343, 2002.

[21] L. Van Der Torre, "Deontic redundancy: a fundamental challenge for deontic logic," *Deontic Logic in Computer Science*, pp. 11–32, 2010.

[22] S. Klarman and V. Gutiérrez-Basulto, "Alcalc: a context description logic," in *Proceedings of the 12th JELIA*, ser. JELIA'10. Berlin, Heidelberg: Springer-Verlag, 2010, pp. 208–220.