O How Kelsenian Jurisprudence and Intuitionistic Logic help to avoid Contrary-to-Duty paradoxes in Legal Ontologies

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Historical Scenario

- Gentzen, G., 19341935, Untersuchungen uber das logische Schliessen (Investigation into Logical Inference), Ph.D. thesis, Universitat Gottingen.
- H Kelsen. Pure theory of law, 1934 (2ed 1960). English Edition 1967.
- KR (Semantic Web) and Proof Theory.
- How Logic is as important as OntoLogy in Knowledge Representation.

What is an Ontology?

- A declarative description of a domain, a Knowledge Base. A set of logical statements that aims do describe a domain completely
- Ontology consistency is mandatory, that is, absence of contradictions
- Negation is an essential operator

What does it means the term "law"?

- What does count as the unit of law? Open question, a.k.a. The individuation problem.
- Joseph Raz. The Concept of a Legal System, 1970.
- What is to count as one complete law, Naturally justified law versus Positive Law.

Positive Law

- According to positivism, law is a matter of what has been posited (ordered, decided, practiced, tolerated, etc.);
- In a more modern idiom, positivism is the view that law is a social construction
- The fact that it might be unjust, unwise, inefficient or imprudent is never sufficient reason for doubting its legality
- Joseph Raz: validity of a law can never depend on its morality

Natural Law

- Can be invoked to criticize judicial decisions about what the law says but not to criticize the best interpretation of the law itself
- Laws are immanent in nature; that is, they can be discovered or found but not created
- Law can emerge by the natural process of resolving conflicts, as embodied by the evolutionary process of the common law

Whereas legal positivism would say that a law can be unjust without it being any less a law, a natural law jurisprudence would say that there is something legally deficient about an unjust law.

Two distinct approaches to the individuation problem

1. Taking all valid statements as in conformance with a declarative statement of an ideal Legally perfect world. This totality is called <u>the law</u>
2. Taking into account all individually legal valid statement as individual laws positively stated and <u>the law</u> is this set

(2) Facilitates the analysis of structural relationship between laws, viz. <u>Primary and Secondary Rules</u> and explicit <u>Grundnorms</u>. Quite adequate to Legal AI.

Why we do not consider Deontic Modal Logic?

- Deontic Logic does not properly distinguish between the normative status of a situation from the normative status of a norm (rule) (Valente 1995)
- Norms should not have truth-value, they are not propositions. (General Theory of Norms, Kelsen 1979/1991)
- An individual law is not a deontic statement, it is not even a proposition. (Kelsen, Alchourrón etc)
- Deontic logic approach to legal knowledge representation brings us paradoxes

Description Logics

FOL$\longmapsto$ Semantic-Network $\longmapsto$ Conceptual-Graphs $\longmapsto$ DLs

- Among the best logical frameworks to represent knowledge
- Binary (Roles) and unary (Concepts) predicate symbols, $R(x, y)$ and $C(y)$.
- Prenex Guarded formulas ($\forall y(R(x, y) \rightarrow C(y))$, $\exists y(R(x, y) \wedge C(y))$) (decidable fragment of FOL).
- Non-trivial extensions (transitive Closure R^*).
- Essentially propositional (Tboxes), but may involve reasoning on individuals (Aboxes).
- ALC can be interpreted as a multi-modal logic $\mathcal{K}$.

$\mathcal{ALC}$ is the core of DLs

- Syntax :

$$
\begin{aligned}
C &::= \bot \mid A \mid \neg C \mid C \sqcap C \mid C \sqcup C \mid \exists R.C \mid \forall R.C \\
F &::= C \sqsubseteq C \mid C \equiv C
\end{aligned}
$$

- Semantics :

$$
\begin{array}{rcl}
\top^{\mathcal{I}} & = & \Delta^{\mathcal{I}} \\
\bot^{\mathcal{I}} & = & \emptyset \\
(\neg C)^{\mathcal{I}} & = & \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
(C \sqcap D)^{\mathcal{I}} & = & C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} & = & C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(\exists R.C)^{\mathcal{I}} & = & \{a \in \Delta^{\mathcal{I}} \mid \exists b.(a, b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\} \\
(\forall R.C)^{\mathcal{I}} & = & \{a \in \Delta^{\mathcal{I}} \mid \forall b.(a, b) \in R^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\} \\
A \sqsubseteq B^{\mathcal{I}} & = & A^{\mathcal{I}} \subseteq B^{\mathcal{I}}
\end{array}
$$

Reasoning Algorithms

- Known proof-procedures (including some industrial Theorem Provers) are based on a specialized FOL Tableaux. Strongly based on individuals even if no ABox is present. (Baader 2003, Horrocks 1998).
- (McGuinees 96) Presented a Sequent Calculus defined from a standard way from the Tableaux. It has been shown to be not so good for explanation extracting.
- Proof Theory for Description Logics, (Rademaker 2010).

A T-Box on Family Relationships using $\mathcal{ALCQ}$

$$
\begin{array}{rcl}
\mathit{Woman} & \equiv & \mathit{Person} \sqcap \mathit{Female} \\
\mathit{Man} & \equiv & \mathit{Person} \sqcap \neg \mathit{Woman} \\
\mathit{Mother} & \equiv & \exists \mathit{hasChild}.\mathit{Person} \sqcap \mathit{Woman} \\
\mathit{Father} & \equiv & \exists \mathit{hasChild}.\mathit{Person} \sqcap \mathit{Man} \\
\mathit{Parent} & \equiv & \mathit{Father} \sqcup \mathit{Mother} \\
\mathit{Grandmother} & \equiv & \mathit{Mother} \sqcap \exists \mathit{hasChild}.\mathit{Parent} \\
\mathit{MotherWithoutDaughter} & \equiv & \mathit{Mother} \sqcap \forall \mathit{hasChild}.\neg \mathit{Woman} \\
\mathit{MotherInTrouble} & \equiv & \mathit{Mother} \sqcap (\geq 10 \mathit{hasChild}).\top
\end{array}
$$

The static part of the trial

- Considering a jurisprudence basis, classical $\mathcal{ALC}$ is not adequate to our approach. We use an intuitionistic version, $i\mathcal{ALC}$
- Dealing with the common (deontic) paradoxes
- A proof-theoretical basis to legal reasoning and explanation
- laws are inhabitants of a universe that must be formalized
- Propositions are about laws and not the laws themselves
- $i\mathcal{ALC}$ was designed to logically support reasoning on Legal Ontologies based on Kelsen jurisprudence
- Defaulf $i\mathcal{ALC}$ is the non-monotonic extension of $i\mathcal{ALC}$ to deal with the dynamics of legal processes (We will not talk about it today!)

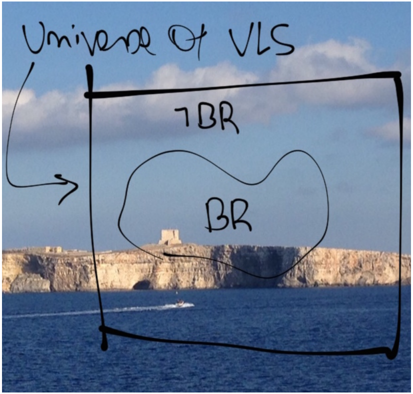
Haeusler, De Paiva, Rademaker (2010-14).
See `http://arademaker.github.io/publications/`

Formalization of a Legal System

- The first-class citizens of any Legal System are VLS. Only VLS inhabit the legal world
- There can be concepts (collections of laws, VLS) and relationships between VLS. For example: PIL (Private International Law), CIVIL, FAMILY etc, can be concepts. LexDomicilium can be a relationship, a.k.a. a legal connection
- The relationships between concepts facilitates the analysis of structural relationships between laws
- The a natural precedence between VLS, e.g. Peter is liable precedes Peter has a renting contract, is modeled as a special relationships between VLS

Intuitionistic vs. Classical Logic (1)

- The extension of an ALC concept is a set
- In Brazil, 18 years-old is a legal age. Let BR contains all VLS in Brazil
- Peter is 17 so Peter is liable is not on BR iff Peter is liable is in the complement of BR
- Classical negation forces the VLS Peter is liable be valid in some legal system outside Brazil



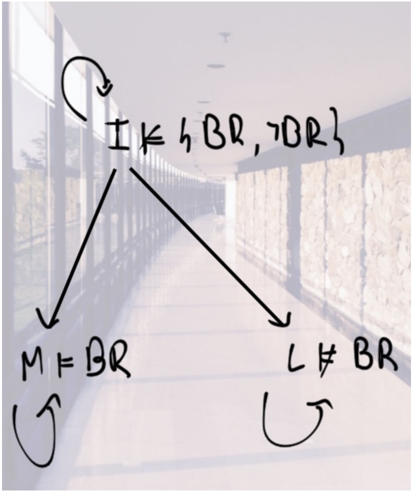
That is, $\phi \sqcup \neg\phi$ is the universe for all ϕ.

Intuitionistic vs. Classical Logic (2)

- We can have neither Peter is liable $\in BR$ nor Peter is liable $\in \neg BR$. Where $pl \in \neg BR$ means
  - $pl : \neg BR$
  - $\mathcal{I}, pl \models \neg BR$
  - $\forall z.z \geq pl$ we have that $z \not\models BR$
- There is no z with $z \geq pl$ such that $\mathcal{I}, z \models BR$. There is no VLS in BR dominating Peter is liable

$\models_i \neg A$, iff, for all j, if $i \preceq j$ then $\not\models_j A$

$\not\models_i \neg\neg A \to A$ and $\not\models_i A \vee \neg A$



Comparing with the deontic logic approach

Deontic approach **Laws** must be taken as **propositions**?, or

iALC/Kelsenian approach **Laws** are inhabitants of a universe that must be formalized, i.e:

Main question

Propositions are about laws or they are the laws themselves?

$i\mathcal{ALC}$: a logic for legal theories formalization

- It can reasoning on individuals (Aboxes), expressed as $i : C$.
- It is not First-order Intuitionistic Logic. It is a genuine Hybrid logic.

$$C, D ::= A \mid \bot \mid \top \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid C \sqsubseteq D \mid \exists R.C \mid \forall R.C$$

A are general assertions and N nominal assertions for ABOX reasoning. Formulas (F) also includes subsumption of concepts interpreted as propositional statements.

$$N ::= x \colon C \mid x \colon N \qquad A ::= N \mid xRy \mid x \leq y \qquad F ::= A \mid C \sqsubseteq D$$

where x and y are nominals, R is a role symbol and C, D are concepts. In particular, this allows $x \colon (y \colon C)$, which is a perfectly valid nominal assertion with x begin its the outer nominal.

$i\mathcal{ALC}$ Semantics

- Semantics is Provided by a structure $\mathcal{I} = (\Delta^{\mathcal{I}}, \preceq^{\mathcal{I}}, \cdot^{\mathcal{I}})$ closed under refinement, i.e., $y \in A^{\mathcal{I}}$ and $x \preceq^{\mathcal{I}} y$ implies $x \in A^{\mathcal{I}}$.
- The interpretation $\mathcal{I}$ is lifted from atomic concepts to arbitrary concepts via:

$$
\begin{array}{rl}
\top^{\mathcal{I}} & =_{df} \Delta^{\mathcal{I}} \\
\bot^{\mathcal{I}} & =_{df} \emptyset \\
(\neg C)^{\mathcal{I}} & =_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}.x \preceq y \Rightarrow y \notin C^{\mathcal{I}}\} \\
(C \sqcap D)^{\mathcal{I}} & =_{df} C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} & =_{df} C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(C \sqsubseteq D)^{\mathcal{I}} & =_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}.(x \preceq y \text{ and } y \in C^{\mathcal{I}}) \Rightarrow y \in D^{\mathcal{I}}\} \\
(\exists R.C)^{\mathcal{I}} & =_{df} \{x \mid \exists y \in \Delta^{\mathcal{I}}.(x, y) \in R^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\} \\
(\forall R.C)^{\mathcal{I}} & =_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}.x \preceq y \Rightarrow \forall z \in \Delta^{\mathcal{I}}.(y, z) \in R^{\mathcal{I}} \Rightarrow z \in C^{\mathcal{I}}\}
\end{array}
$$

Restrictions on the Interpretations

The structures $\mathcal{I}$ are models for $i\mathcal{ALC}$ they satisfy two frame conditions:

- F1 if $w \leq w'$ and wRv then $\exists v'.w'Rv'$ and $v \leq v'$
- F2 if $v \leq v'$ and wRv then $\exists w'.w'Rv'$ and $w \leq w'$

The above conditions are diagrammatically expressed as:

$$\begin{tikzcd} w' \arrow[r, dotted, "R"] & v' \\ w \arrow[u, "\leq"] \arrow[r, "R"] \arrow[ur, phantom, "(F1)"] & v \arrow[u, dotted, "\leq"'] \end{tikzcd} \quad \text{and} \quad \begin{tikzcd} w' \arrow[r, dotted, "R"] & v' \\ w \arrow[u, dotted, "\leq"] \arrow[r, "R"] \arrow[ur, phantom, "(F2)"] & v \arrow[u, "\leq"'] \end{tikzcd}$$

Contrary-to-Duty (or Chisholm's 1963) Paradox

1. It ought to be that Jones goes to the assistance of his neighbors.
2. It ought to be that if Jones does go then he tells them he is coming.
3. If Jones doesn't go, then he ought not tell them he is coming.
4. Jones doesn't go.

- This certainly appears to describe a possible situation. 1-4 constitute a mutually consistent and logically independent set of sentences.
- (1) is a primary obligation, what Jones ought to do unconditionally. (2) is a compatible-with-duty obligation, appearing to say (in the context of 1) what else Jones ought to do on the condition that Jones fulfills his primary obligation. (3) is a contrary-to-duty obligation (CTD) appearing to say (in the context of 1) what Jones ought to do conditional on his violating his primary obligation. (4) is a factual claim, which conjoined with (1), implies that Jones violates his primary obligation.

Standard Deontic Logic (SDL)

The axioms of SDL:

TAUT	all tautologies wffs of the language
OB-K	$O(p \to q) \to (Op \to Oq)$
OB-D	$Op \to \neg O \neg p$
MP	if $\vdash p$ and $\vdash p \to q$ then $\vdash q$
OB-NEC	if $\vdash p$ then $\vdash Op$

SDL is just the normal modal logic D or KD, with a suggestive notation expressing the intended interpretation.

From these, we can prove the principle that obligations cannot conflict, NC of SDL, $\neg(Op \wedge O\neg p)$.

Contrary-to-Duty Paradox in SDL

1. Op
2. $O(p \to q)$
3. $\neg p \to O\neg q$
4. $\neg p$

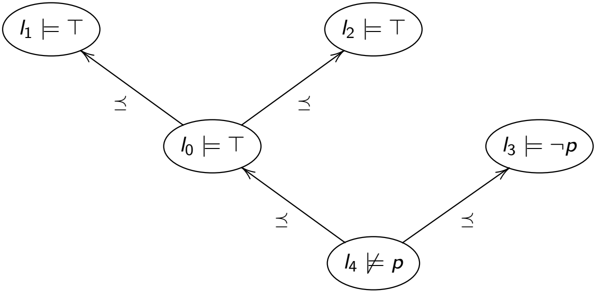
But Chisholm points out

- from (2) by principle OB-K we get $Op \to Oq$,
- and then from (1) by MP, we get Oq;
- but by MP alone we get $O\neg q$ from (3) and (4).
- From these two conclusions, by PC, we get $Oq \wedge O\neg q$, contradicting NC of SDL.

Thus 1-4 leads to inconsistency per SDL. But 1-4 do not seem inconsistent at all, so the representation cannot be a faithful one.

An *iALC* model for the Chisholm (ex) paradox

1. The law l_1, originally Op
2. The law l_2, originally $O(p \rightarrow q)$
3. From (3), $\neg p \rightarrow O\neg q$, we have $l_3 : \neg p$. If we had $O\neg q \rightarrow \neg p$ the translation would be the same. That is, l_3 is $O\neg q$.
4. The law l_0 that represents the infinum of l_1 and l_2.



Remember that if $x : A$ then $\forall x' \geq x, x' : A$.

Summary of the Approach

- Individual Legal Valid Statements are the individuals of the universe.
- Concepts are Classes of individual laws.
- Roles (relationships) between individuals laws denote kinds of Legal Connections
- Subsumptions and Negations are intuitionistically interpreted ($iALC$)

Conclusions

- Using $\mathcal{ALC}$ instead of $i\mathcal{ALC}$ seems to
  - lead us considering a legal ontology involving non-valid Legal Statements
  - deal with ad hoc ontology regarding jurisprudence main concepts.
  - increase complexity, since many non-valid Legal Statements might have to be considered.
- Adequate according philosophical and jurisprudence theory.
- Juridic cases can be analyzed in the ABOX.
- TBOX describes "The Law".
- There is a Deductive System for $i\mathcal{ALC}$, the logic is decidable.
- $\preceq$ is not always specified at the level of the TBOX.
- It seems to scale, but there is no empirical evidence. Is the coherence analysis easier? Work out "hard juridical cases".
- Can be the kernel of a tool for helping with a judge's decision (not a sentence writer!)

Extra slides if we have time for them!

A Sequent Calculus for **iALC**

$$\frac{}{\Delta, \delta \Rightarrow \delta} \qquad\qquad \frac{}{\Delta, x\colon \bot \Rightarrow \delta}$$

$$\frac{\Delta, xRy \Rightarrow y\colon \alpha}{\Delta \Rightarrow x\colon \forall R.\alpha}\ \forall\text{-r} \qquad\qquad \frac{\Delta, x\colon \forall R.\alpha, y\colon \alpha, xRy \Rightarrow \delta}{\Delta, x\colon \forall R.\alpha, xRy \Rightarrow \delta}\ \forall\text{-l}$$

$$\frac{\Delta \Rightarrow xRy \qquad \Delta \Rightarrow y\colon \alpha}{\Delta \Rightarrow x\colon \exists R.\alpha}\ \exists\text{-r} \qquad\qquad \frac{\Delta, xRy, y\colon \alpha \Rightarrow \delta}{\Delta, x\colon \exists R.\alpha \Rightarrow \delta}\ \exists\text{-l}$$

$$\frac{\Delta, \alpha \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqsubseteq \beta}\ \sqsubseteq\text{-r} \qquad\qquad \frac{\Delta_1 \Rightarrow \alpha \qquad \Delta_2, \beta \Rightarrow \delta}{\Delta_1, \Delta_2, \alpha \sqsubseteq \beta \Rightarrow \delta}\ \sqsubseteq\text{-l}$$

$$\frac{\Delta \Rightarrow \alpha \qquad \Delta \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqcap \beta}\ \sqcap\text{-r} \qquad\qquad \frac{\Delta, \alpha, \beta \Rightarrow \delta}{\Delta, \alpha \sqcap \beta \Rightarrow \delta}\ \sqcap\text{-l}$$

$$\frac{\Delta \Rightarrow \alpha}{\Delta \Rightarrow \alpha \sqcup \beta}\ \sqcup_1\text{-r} \qquad\qquad \frac{\Delta, \alpha \Rightarrow \delta \qquad \Delta, \beta \Rightarrow \delta}{\Delta, \alpha \sqcup \beta \Rightarrow \delta}\ \sqcup\text{-l}$$

$$\frac{\Delta, \alpha \Rightarrow \beta}{\forall R.\Delta, \exists R.\alpha \Rightarrow \exists R.\beta}\ \text{p-}\exists \qquad\qquad \frac{\Delta \Rightarrow \alpha}{\forall R.\Delta \Rightarrow \forall R.\alpha}\ \text{p-}\forall$$

$$\frac{\Delta \Rightarrow \delta}{x\colon \Delta \Rightarrow x\colon \delta}\ \text{p-N}$$

All propositional rules have their nominal version.

Intuitionistic interpretation of a sequent

- The semantics of the sequent $\Theta, \Gamma \Rightarrow \delta$ is $\Theta, \Gamma \models \delta$.
- We write $\Theta, \Gamma \models \delta$ if it is the case that:

$$\forall \mathcal{I}.((\forall x.\, \mathcal{I}, x \models \Theta) \Rightarrow \forall \vec{z} \succeq \mathit{Nom}(\Gamma, \delta).(\mathcal{I}, \vec{z} \models \Gamma \Rightarrow \mathcal{I}, \vec{z} \models \delta)$$

  where $\vec{z}$ denotes a vector of variables $z_1, \ldots, z_k$ and $\mathit{Nom}(\Gamma, \delta)$ is the vector of all outer nominals occurring in each nominal assertion of $\Gamma \cup \{\delta\}$. x is the only outer nominal of a nominal assertion $\{x : \gamma\}$, while a (pure) concept γ has no outer nominal.

Using **iALC** to formalize Conflict of Laws in Space

Peter and Maria signed a renting contract. The subject of the contract is an apartment in Rio de Janeiro. The contract states that any dispute will go to court in Rio de Janeiro. Peter is 17 and Maria is 21. Peter lives in Edinburgh and Maria lives in Rio.

Only legally capable individuals have civil obligations:

$PeterLiable \preceq ContractHolds@RioCourt$, shortly, $pl \preceq cmp$
$MariaLiable \preceq ContractHolds@RioCourt$, shortly, $ml \preceq cmp$

Concepts, nominals and their relationships:

BR is the collection of Brazilian Valid Legal Statements
SC is the collection of Scottish Valid Legal Statements
PIL_{BR} is the collection of Private International Laws in Brazil
$ABROAD$ is the collection of VLS outside Brazil
$LexDomicilium$ is a legal connection: the pair $\langle pl, pl \rangle$ is in $LexDomicilium$

Non-Logical Axiom Sequents

The sets Δ, of concepts, and Ω, of **iALC** sequents representing the knowledge about the case.

$$\Delta = \boxed{\begin{array}{ccc} ml : \mathrm{BR} & pl : \mathrm{SC} & pl \preceq cmp \\ ml \preceq cmp & pl \ \mathrm{LexDom} \ pl & \end{array}}$$

$$\Omega = \boxed{\begin{array}{c} \mathrm{PIL_{BR}} \Rightarrow \mathrm{BR} \\ \mathrm{SC} \Rightarrow \mathrm{ABROAD} \\ \exists \mathrm{LexD_1.L_1} \ldots \sqcup \exists \mathrm{LexDom.ABROAD} \sqcup \ldots \exists \mathrm{LexD_k.L_k} \Rightarrow \mathrm{PIL_{BR}} \end{array}}$$

A proof in our SC

$$
\dfrac{
  \dfrac{
    \dfrac{
      \Delta \Rightarrow \mathit{pl} : \mathsf{SC}
      \qquad
      \dfrac{\Omega}{\mathit{pl} : \mathsf{SC} \Rightarrow \mathit{pl} : \mathsf{A}}
    }{
      \Delta \Rightarrow \mathit{pl} : \mathsf{A}
    }\;\mathit{cut}
    \qquad
    \Delta \Rightarrow \mathit{pl}\ \mathsf{LexD}\ \mathit{pl}
  }{
    \Delta \Rightarrow \mathit{pl} : \exists \mathsf{LexD.A}
  }\;\exists\text{-R}
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{\exists \mathsf{LexD.A} \Rightarrow \exists \mathsf{LexD.A}}{\exists \mathsf{LexD.A} \Rightarrow \mathsf{PIL_{BR}}}\;\sqcup\text{-R}
      \qquad
      \dfrac{\Omega}{\mathsf{PIL_{BR}} \Rightarrow \mathsf{BR}}
    }{
      \exists \mathsf{LexD.A} \Rightarrow \mathsf{BR}
    }\;\text{cut}
  }{
    \mathit{pl} : \exists \mathit{LexD.A} \Rightarrow \mathit{pl} : \mathit{BR}
  }\;\text{p-N}
}{
  \Delta \Rightarrow \mathit{pl} : \mathsf{BR}
}\;\text{cut}
$$

$$
\dfrac{
  \Delta \Rightarrow \mathit{ml} : \mathsf{BR}
  \qquad
  \dfrac{
    \dfrac{\Pi}{\Delta \Rightarrow \mathit{pl} : \mathsf{BR}}
    \qquad
    \dfrac{\Omega}{\mathit{ml} : \mathsf{BR}, \mathit{pl} : \mathsf{BR} \Rightarrow \mathit{cmp} : \mathsf{BR}}
  }{
    \Delta, \mathit{ml} : \mathsf{BR} \Rightarrow \mathit{cmp} : \mathsf{BR}
  }\;\mathit{cut}
}{
  \Delta \Rightarrow \mathit{cmp} : \mathsf{BR}
}\;\mathit{cut}
$$

Metatheorems

- $i\mathcal{ALC}$ is sound and complete regarded Intuitionistic Conceptual Models (Hylo 2010)
- $IPL \subset i\mathcal{ALC}$ (hardness is PSPACE)
- Alternating Polynomial Turing-Machine to find out winner-strategy on the SAT-Game of a hybrid language. (upper-bound is PSPACE).

SAT in **iALC** $\subset$ PSPACE

- One wants fo verify whether $\Theta, \Gamma \Rightarrow \gamma$ is satisfiable.
- $\Theta, \Gamma \Rightarrow \gamma$ is satisfiable, if and only if, $(\sqcap_{\theta \in \Theta} \theta) \sqsubseteq \gamma$ is satisfiable in a model of Γ. A game is defined on $\Gamma \cup \{\xi\}$
- $\exists$*loise* starts by playing a list $\{H_0, \ldots, H_k\}$ of $\Gamma \cup \{\xi\}$ of Hintikka I-sets, and two relations $\mathcal{R}$ and $\preccurlyeq$ on them.
- $\exists$*loise* loses if she cannot provide the list as a pre-model.
- $\forall$*belard* chooses a set from the list and a formula inside this set.
- $\exists$*loise* has to fulfill extend the (pre)-model in order to satisfy the formula.
- $\Gamma \cup \xi$ is satisfiable, iff, $\exists$*loise* has a winning strategy.