An Intuitionistic **ALC** Description Default Logic

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What does it mean the term “law”?

- Two main (distinct) approaches! Open question, a.k.a. “The individuation problem”.
- (Raz 1972) What is to count as one “complete law”? Taking all (existing) legally valid statements as a whole. This totality is called “the law”, a perfect legal world.
- Legal Positivism tradition (Kelsen 1934). Question: Natural coherence versus Knowledge Management resulted coherence. Taking into account all individual legally valid statement as individual laws.
- The second seems to be quite adequate to Legal AI. Facilitates the analysis of structural relationship between laws, viz. Primary and Secondary Rules.

Basic Motivations

- Description Logic is among the best logical frameworks to represent knowledge.
- Powerful language expression and decidable.
- In legal situations, we usually draw default conclusions, based on our experience and on what is called typical, which may be refutable in face of new knowledge (nonmonotonic reasoning, defaults).

Our approach: the (static) part of a trial

- Considering a jurisprudence basis, classical **ALC** it is not adequate to our approach. We use an intuitionistic version, **iALC**.
- Dealing with the common (deontic) paradoxes.
- A proof-theoretical basis to legal reasoning and explanation.
- **laws** are inhabitants of a universe that must be formalized.
- <u>Propositions are about laws</u> and not the laws themselves.

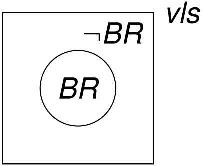
Haeusler, De Paiva, Rademaker (2010-2011).

Formalization of a Legal System

- The first-class citizens of any Legal System are vls. Only vls inhabit the legal world.
- There can be concepts (collections of laws) on vls and relationships between vls. For example: PIL_{BR}, *CIVIL*, *FAMILY*, etc, can be concepts. *LexDomicilium* can be a relationship, a.k.a. a legal connection.
- The relationships between concepts facilitates the analysis of structural relationships between laws.
- The natural precedence between laws, e.g. " Peter is liable" precedes "Peter has a renting contract", is modeled as a special relationships between laws.

Intuitionistic versus Classical logic

- The extension of an **ALC** concept is a **Set**.



- Classical Negation: $\neg\phi \vee \phi$ is valid for any ϕ.

In *BR*, 18 is the legal age *BR* contains all vls in Brazil

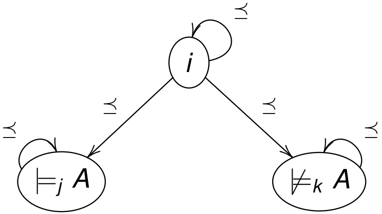
"Peter is 17"

"Peter is liable" $\notin$ *BR* iff "Peter is liable" $\in \neg$*BR*

- Classical negation forces the "Peter is liable" be valid in some legal system outside Brazil.

Intuitionistic versus Classical logic (cont.)

- The Intuitionistic Negation $\models_i \neg A$, iff, for all j, if $i \preceq j$ then $\not\models_j A$



$$\not\models_i \neg\neg A \rightarrow A \quad \text{and} \quad \not\models_i A \vee \neg A$$

- In an intuitionistically based approach to Law, we can have neither "Peter is liable"$\notin BR$ nor "Peter is liable"$\in \neg BR$.
- $pl \in \neg BR$ means $pl : \neg BR$ means $\mathcal{I}, pl \models \neg BR$ or $\forall z.\ z \succeq pl$ we have $z \not\models BR$.
- In other words, there is no z with $z \succeq pl$ such that $\mathcal{I}, z \models BR$. There is no <u>vls</u> in BR dominating "Peter is liable".

The logical for legal theories formalization

- Binary (Roles) and unary (Concepts) predicate symbols, $R(x, y)$ and $C(y)$.
- It is not First-order Intuitionistic Logic. It is a genuine Hybrid logic.

$$C, D ::= A \mid \bot \mid \top \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid C \sqsubseteq D \mid \exists R.C \mid \forall R.C$$

A are general assertions and N nominal assertions for ABOX reasoning. Formulas (F) also includes subsumption of concepts interpreted as propositional statements.

$$N ::= x : C \mid x : N \qquad A ::= N \mid xRy \mid x \leq y \qquad F ::= A \mid C \sqsubseteq D$$

where x and y are nominals, R is a role symbol and C, D are concepts.

Interpretation

- Semantics is Provided by a structure $\mathcal{I} = (\Delta^{\mathcal{I}}, \preceq^{\mathcal{I}}, \cdot^{\mathcal{I}})$ closed under refinement, i.e., $y \in A^{\mathcal{I}}$ and $x \preceq^{\mathcal{I}} y$ implies $x \in A^{\mathcal{I}}$.
- The interpretation $\mathcal{I}$ is lifted from atomic concepts to arbitrary concepts via:

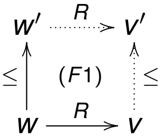
$$
\begin{aligned}
\top^{\mathcal{I}} &=_{df} \Delta^{\mathcal{I}} \\
\bot^{\mathcal{I}} &=_{df} \emptyset \\
(\neg C)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}.x \preceq y \Rightarrow y \notin C^{\mathcal{I}}\} \\
(C \sqcap D)^{\mathcal{I}} &=_{df} C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} &=_{df} C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(C \sqsubseteq D)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}.(x \preceq y \text{ and } y \in C^{\mathcal{I}}) \Rightarrow y \in D^{\mathcal{I}}\} \\
(\exists R.C)^{\mathcal{I}} &=_{df} \{x \mid \exists y \in \Delta^{\mathcal{I}}.(x, y) \in R^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\} \\
(\forall R.C)^{\mathcal{I}} &=_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}.x \preceq y \Rightarrow \forall z \in \Delta^{\mathcal{I}}.(y, z) \in R^{\mathcal{I}} \Rightarrow z \in C^{\mathcal{I}}\}
\end{aligned}
$$

Restrictions on the Interpretations

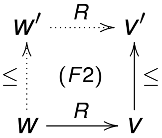
Following the semantics of IK, the structures $\mathcal{I}$ are models for **iALC** if they satisfy two frame conditions:

- **F1** if $w \leq w'$ and wRv then $\exists v'.w'Rv'$ and $v \leq v'$
- **F2** if $v \leq v'$ and wRv then $\exists w'.w'Rv'$ and $w \leq w'$

The above conditions are diagrammatically expressed as:



and



A Sequent Calculus for **iALC**

$$\frac{}{\Delta, \delta \Rightarrow \delta} \qquad \frac{}{\Delta, x: \bot \Rightarrow \delta}$$

$$\frac{\Delta, xRy \Rightarrow y: \alpha}{\Delta \Rightarrow x: \forall R.\alpha}\ \forall\text{-r} \qquad \frac{\Delta, x: \forall R.\alpha, y: \alpha, xRy \Rightarrow \delta}{\Delta, x: \forall R.\alpha, xRy \Rightarrow \delta}\ \forall\text{-l}$$

$$\frac{\Delta \Rightarrow xRy \qquad \Delta \Rightarrow y: \alpha}{\Delta \Rightarrow x: \exists R.\alpha}\ \exists\text{-r} \qquad \frac{\Delta, xRy, y: \alpha \Rightarrow \delta}{\Delta, x: \exists R.\alpha \Rightarrow \delta}\ \exists\text{-l}$$

$$\frac{\Delta, \alpha \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqsubseteq \beta}\ \sqsubseteq\text{-r} \qquad \frac{\Delta_1 \Rightarrow \alpha \qquad \Delta_2, \beta \Rightarrow \delta}{\Delta_1, \Delta_2, \alpha \sqsubseteq \beta \Rightarrow \delta}\ \sqsubseteq\text{-l}$$

$$\frac{\Delta \Rightarrow \alpha \qquad \Delta \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqcap \beta}\ \sqcap\text{-r} \qquad \frac{\Delta, \alpha, \beta \Rightarrow \delta}{\Delta, \alpha \sqcap \beta \Rightarrow \delta}\ \sqcap\text{-l}$$

$$\frac{\Delta \Rightarrow \alpha}{\Delta \Rightarrow \alpha \sqcup \beta}\ \sqcup_1\text{-r} \qquad \frac{\Delta, \alpha \Rightarrow \delta \qquad \Delta, \beta \Rightarrow \delta}{\Delta, \alpha \sqcup \beta \Rightarrow \delta}\ \sqcup\text{-l}$$

$$\frac{\Delta, \alpha \Rightarrow \beta}{\forall R.\Delta, \exists R.\alpha \Rightarrow \exists R.\beta}\ \text{p-}\exists \qquad \frac{\Delta \Rightarrow \alpha}{\forall R.\Delta \Rightarrow \forall R.\alpha}\ \text{p-}\forall$$

$$\frac{\Delta \Rightarrow \delta}{x: \Delta \Rightarrow x: \delta}\ \text{p-N}$$

All propositional rules have their nominal version.

Intuitionistic interpretation of a sequent

- The semantics of the sequent $\Theta, \Gamma \Rightarrow \delta$ is $\Theta, \Gamma \models \delta$.
- We write $\Theta, \Gamma \models \delta$ if it is the case that:

$$\forall \mathcal{I}.((\forall x.\ \mathcal{I}, x \models \Theta) \Rightarrow \forall \vec{z} \succeq Nom(\Gamma, \delta).(\mathcal{I}, \vec{z} \models \Gamma \Rightarrow \mathcal{I}, \vec{z} \models \delta)$$

where $\vec{z}$ denotes a vector of variables $z_1, \ldots, z_k$ and $Nom(\Gamma, \delta)$ is the vector of all outer nominals occurring in each nominal assertion of $\Gamma \cup \{\delta\}$. x is the only outer nominal of a nominal assertion $\{x : \gamma\}$, while a (pure) concept γ has no outer nominal.

Using **iALC** to formalize Conflict of Laws in Space

Peter and Maria signed a renting contract. The subject of the contract is an apartment in Rio de Janeiro. The contract states that any dispute will go to court in Rio de Janeiro. Peter is 17 and Maria is 20. Peter lives in Edinburgh and Maria lives in Rio.

Only legally capable individuals have civil obligations:

$PeterLiable \preceq ContractHolds@RioCourt$, shortly, $pl \preceq cmp$
$MariaLiable \preceq ContractHolds@RioCourt$, shortly, $ml \preceq cmp$

Concepts, nominals and their relationships:

BR is the collection of Brazilian Valid Legal Statements
SC is the collection of Scottish Valid Legal Statements
PIL_{BR} is the collection of Private International Laws in Brazil
$ABROAD$ is the collection of VLS outside Brazil
$LexDomicilium$ is a legal connection: the pair $\langle pl, pl \rangle$ is in $LexDomicilium$

Non-Logical Axiom Sequents

The sets Δ, of concepts, and Ω, of **iALC** sequents representing the knowledge about the case.

$$\Delta = \boxed{\begin{array}{ccc} ml : \mathrm{BR} & pl : \mathrm{SC} & pl \preceq cmp \\ ml \preceq cmp & pl\ \mathrm{LexDom}\ pl & \end{array}}$$

$$\Omega = \boxed{\begin{array}{c} \mathrm{PIL_{BR}} \Rightarrow \mathrm{BR} \\ \mathrm{SC} \Rightarrow \mathrm{ABROAD} \\ \exists \mathrm{LexD_1.L_1} \ldots \sqcup \exists \mathrm{LexDom.ABROAD} \sqcup \ldots \exists \mathrm{LexD_k.L_k} \Rightarrow \mathrm{PIL_{BR}} \end{array}}$$

A proof in our SC

$$
\dfrac{
\dfrac{
\dfrac{\Delta \Rightarrow pl : \mathrm{SC} \qquad \dfrac{\Omega}{pl : \mathrm{SC} \Rightarrow pl : \mathrm{A}}}{\Delta \Rightarrow pl : \mathrm{A}}\,\mathit{cut}
\qquad \Delta \Rightarrow pl\ \mathrm{LexD}\ pl
}{\Delta \Rightarrow pl : \exists \mathrm{LexD.A}}\,\exists\text{-R}
\qquad
\dfrac{
\dfrac{
\dfrac{\exists \mathrm{LexD.A} \Rightarrow \exists \mathrm{LexD.A}}{\exists \mathrm{LexD.A} \Rightarrow \mathrm{PIL_{BR}}}\,\sqcup\text{-R}
\qquad
\dfrac{\Omega}{\mathrm{PIL_{BR}} \Rightarrow \mathrm{BR}}
}{\exists \mathrm{LexD.A} \Rightarrow \mathrm{BR}}\,\text{cut}
}{pl : \exists \mathit{LexD.A} \Rightarrow pl : \mathit{BR}}\,\text{p-N}
}{\Delta \Rightarrow pl : \mathrm{BR}}\,\text{cut}
$$

$$
\dfrac{
\Delta \Rightarrow ml : \mathrm{BR}
\qquad
\dfrac{
\dfrac{\Pi}{\Delta \Rightarrow pl : \mathrm{BR}}
\qquad
\dfrac{\Omega}{ml : \mathrm{BR}, pl : \mathrm{BR} \Rightarrow cmp : \mathrm{BR}}
}{\Delta, ml : \mathrm{BR} \Rightarrow cmp : \mathrm{BR}}\,\mathit{cut}
}{\Delta \Rightarrow cmp : \mathrm{BR}}\,\mathit{cut}
$$

Soundness

Proposition

If $\Theta, \Gamma \Rightarrow \delta$ *is provable in* $SC_{\mathbf{iALC}}$ *then* $\Theta, \Gamma \models \gamma$.

Proof: We prove that each sequent rule preserves the validity of the sequent and that the initial sequents are valid.
Note: for each rule, we can derive the soundness of its non-nominal version from the proof of soundness of its nominal version.

Completeness

We shown the relative completeness of $SC_{\mathbf{iALC}}$ regarding the axiomatic presentation (Plotkin 1986, Simpson 95,Fisher 1984):

0.	all substitution instances of theorems of IPL
1.	$\forall R.(C \sqsubseteq D) \sqsubseteq (\forall R.C \sqsubseteq \forall R.D)$
2.	$\exists R.(C \sqsubseteq D) \sqsubseteq (\exists R.C \sqsubseteq \exists R.D)$
3.	$\exists R.(C \sqcup D) \sqsubseteq (\exists R.C \sqcup \exists R.D)$
4.	$\exists R.\bot \sqsubseteq \bot$
5.	$(\exists R.C \sqsubseteq \forall R.C) \sqsubseteq \forall R.(C \sqsubseteq D)$
MP	If C and $C \sqsubseteq D$ are theorems, D is a theorem too.
Nec	If C is a theorem then $\forall R.C$ is a theorem too.

It is sufficient to derive in $SC_{\mathbf{iALC}}$ the axioms 1–5. All instances of IPL theorems can also be proved using only propositional rules. The MP rule is a derived rule using the cut rule. The Nec rule is the p-$\forall$ rule in the system with Δ empty.

Our approach: the dynamic part of a trial

- **iALC** takes care of the task of sentence justification when a set of laws is considered.
- However, in a lawsuit, there are steps that take care of validating which are the valid legal statements to be considered.
- The agents act in a default based way, since the process of including valid legal statements require to verify the (possibly) validity of other legal statements before accepting a (new) valid legal statement.
- $DD_{\mathbf{iALC}}$ is motivated by the aim of providing a logical tool to be used with this purpose.

Example of default reasoning is the case of default judgement of the U.S. Civil Law that states that either party that fails to take action by the other party is sentenced.

Revising previous example with Defaults

- The sets Δ, of concepts, and Ω, of **iALC** sequents representing the knowledge about the case.

$$\Delta = \boxed{\begin{array}{lll} ml : \mathrm{BR} & & pl \preceq cmp \\ ml \preceq cmp & pl \ \mathrm{LexDom}\ pl & \end{array}}$$

$$\Omega = \boxed{\begin{array}{c} \mathrm{PIL_{BR}} \Rightarrow \mathrm{BR} \\ \mathrm{SC} \Rightarrow \mathrm{ABROAD} \\ \exists \mathrm{LexD_1.L_1} \ldots \sqcup \exists \mathrm{LexDom.ABROAD} \sqcup \ldots \exists \mathrm{LexD_k.L_k} \Rightarrow \mathrm{PIL_{BR}} \end{array}}$$

- With one additional default rule:

$$\frac{:\ pl : SC, pid : \neg SC}{pl : SC}$$

where pid means “peter is indebted” (he haven’t payed the income tax).

The system $DD_{\mathbf{iALC}}$

- The decidability to the nonmonotonic reasoning is preserved by imposing an order on the application of default rules that respects the Exceptions-First Principle (Pequeno 1994, Frota 2011).
- We also keep into account the knowledge represented by the description logic **iALC**.
- This principle is a key to solve the anomalous extension problem as the one introduced by the Yale Shooting Problem (Hanks 87).

Towards a constructive definition of extension

Definition (Generating Defaults)

The set of generating defaults GD for E with respect to $\langle W, \mathcal{D} \rangle$ is defined as

$$GD(E, \langle W, \mathcal{D} \rangle) = \{\frac{\alpha : \beta, \gamma}{\beta} \in \mathcal{D} | \alpha \in E, \neg\beta \notin E, \neg\gamma \notin E\}.$$

Definition (Consequents)

If $\mathcal{D}$ is a set of defaults, then $Consequents(\mathcal{D}) = \{\beta | \frac{\alpha:\beta,\gamma}{\beta} \in \mathcal{D}\}$.

Conjecture (Definition of a $DD_{\mathbf{iALC}}$ Extension)

If E is an extension for $\langle W, \mathcal{D} \rangle$, then
$E = Cn(W \cup Consequents(GD(E, \langle W, \mathcal{D} \rangle)))$.

THANK YOU

Ordering relations: $\underline{\ll}$ and $\ll$

Let $\langle W, \mathcal{D}\rangle$ be a description default theory. Without loss of generality, assume that all formulas in W are in clausal form. The partial relations $\underline{\ll}$ and $\ll$ on literals are defined as follows:

1. If $\alpha \in$ W, then $\alpha = \alpha_1 \vee \ldots \vee \alpha_n$, $n \geq 1$. For all $\alpha_i, \alpha_j \in \{\alpha_1, \ldots, \alpha_n\}$, if $\alpha_i \neq \alpha_j$, let $\neg\alpha_i \underline{\ll} \alpha_j$.
2. If $\delta \in \mathcal{D}$, then $\delta = \frac{\alpha:\beta,\gamma}{\beta}$, Let $\alpha_1, \ldots, \alpha_r$, $\beta_1, \ldots, \beta_s$, and $\gamma_1, \ldots, \gamma_t$ be the literals of the $\mathcal{ALC}$ clausal forms of α, β, and γ, respectively. Then:

   2.1 If $\alpha_i \in \{\alpha_1, \ldots, \alpha_r\}$ and $\beta_j \in \{\beta_1, \ldots, \beta_s\}$, let $\alpha_i \underline{\ll} \beta_j$.
   2.2 If $\gamma_i \in \{\gamma_1, \ldots, \gamma_t\}$ and $\beta_j \in \{\beta_1, \ldots, \beta_s\}$, let $\neg\gamma_i \ll \beta_j$.
   2.3 Let $\beta = \beta_1 \wedge \ldots \wedge \beta_m$, for some $m \geq 1$ and, for each $i \leq m$, $\beta_i = (\beta_{i,1} \vee \ldots \vee \beta_{i,m_i})$, such that $m_i \geq 1$. Thus, if $\beta_{i,j}, \ \beta_{i,k} \in \{\beta_{1,1}, \ldots, \beta_{m,m_m}\}$ and $\beta_{i,j} \neq \beta_{i,k}$, then $\neg\beta_{i,j} \underline{\ll} \beta_{i,k}$.

3. The following relationships holds for $\underline{\ll}$ and $\ll$:

   3.1 If $\alpha \underline{\ll} \beta$ and $\beta \underline{\ll} \gamma$, then $\alpha \underline{\ll} \gamma$.
   3.2 If $\alpha \ll \beta$ and $\beta \ll \gamma$, then $\alpha \ll \gamma$.
   3.3 If ($\alpha \ll \beta$ and $\beta \underline{\ll} \gamma$) or ($\alpha \underline{\ll} \beta$ and $\beta \ll \gamma$) then $\alpha \ll \gamma$.

Coherence and Default Partition

Theorem (Coherence)

An ordered description default theory has at least one extension.

Lemma (Default Partition $\{\mathcal{D}_i\}$)

For each ordered description default theory, there is a partition $\{\mathcal{D}_i\}$ for $\mathcal{D}$ defined as follows: For each $\delta = \frac{\alpha:\beta,\gamma}{\beta} \in \mathcal{D}$, we have that $l_{MIN}(\beta) = i \Leftrightarrow \delta \in \mathcal{D}_i$.

Constructive Extension theorem

Let $\langle W, \mathcal{D} \rangle$ be an ordered description default theory and $\{\mathcal{D}_i\}$ a default partition. Let E_0 be an extension to $\langle W, \mathcal{D}_0 \rangle$. For $i > 0$, we construct $\mathcal{D}_{i'} = \{\frac{\alpha:\beta}{\beta} | \frac{\alpha:\beta}{\beta} \in \mathcal{D}_i$, or $\frac{\alpha:\beta,\gamma}{\beta} \in \mathcal{D}_i$ and $\neg\gamma \notin E_{i-1}\}$.
Since $\langle W, \mathcal{D}_{i'} \rangle$ is a normal default theory, it has at least one extension E_i. Hence $E = \bigcup_{i=0}^{\infty} E_i$ is a extension for $\langle W, \mathcal{D} \rangle$.