An Intuitionistic $\mathcal{ALC}$ Description Default Logic

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The knowledge formalization and reasoning automation are central within Artificial Intelligence (AI) investigations. In particular, logical systems have been used as an extremely attractive tool, not only for providing a formal language which allows a non ambiguous knowledge representation, but also for suggesting a precise and powerful knowledge manipulation, the logical inferences.

The classical logic has been traditionally used in AI. However, the classical logic is better suited to model ideal situations in which the knowledge at hand is precise and complete enough to perform inferences whose conclusions have a status of certainty (modulo the assumed premises). In real situations, in which the knowledge is incomplete and imprecise, the classical logic is not sufficient. Non classical logics, as the nonmonotonic ones, have been proposed to better cope with practical reasoning in partial knowledge.

Nonmonotonic logics [6] are logical systems in which the monotonicity property fails. This property assures that, if a conclusion is drawn on the basis of some premises, no additional premise will invalidate this conclusion. However, in real situations, it is usual to draw default conclusions, based on our experience and on what we think as typical, which may be refutable in face of new knowledge.

Reasoning with defaults was formalized by Reiter [14]. Reiter's default logic extends classical first-order logic with default rules, non-standard rules of inference which have a proviso to their application based on the absence of knowledge.

Although powerful enough to formalize default reasoning, Default Logic is undecidable. In order to apply a default rule like $\frac{\alpha(x) : \beta(x)}{\gamma(x)}$, we have to first check if $\neg\beta(x)$ is not derived. Since the set of theorems in classical first-order logic is recursively enumerable, the set of theorems in Default Logic is not even recursively enumerable.

Considering that we are interested in formalizing and reasoning with legal knowledge or legal ontologies, we should design a decidable default logic for such purposes. For that, instead of using classical first-order logic, we use the

intuitionistic description logic $i\mathcal{ALC}$ as the monotonic basis of the default logics $\mathsf{DD}_{i\mathcal{ALC}}$. $\mathsf{DD}_{i\mathcal{ALC}}$ is based on its classical counterpart $\mathsf{DD}_{\mathcal{ALC}}$ [5].

In [9, 7] we discuss how Kelsen's ([12]) pure theory of law points out a framework that takes into account the legal knowledge forming a collection of individual legally valid statements. Thus, each legally valid statement may be seen as an inhabitant among the many individual laws of the represented legal system. The natural precedence existing between individual legal statements can be taken as a pre-order relation on the legal statements. The legal principle that rules the stability of the law implies that the precedence of individual laws preserve properties (decisions, conditions of applicability, adequate fora, etc) regarding them. From this jurisprudence considerations we have that intuitionistic logic is the best choice for reasoning on laws within a non-deontic approach. In [9, 7] it is shown, by means of an example, how the intuitionistic negation is better than its classic counterpart for reasoning with conflict of laws in space in a case regarding Private International Law.

In the presence of this natural precedence order between legally valid statements, the intuitionistic interpretation of subsumption between concepts A and B ($A \subseteq B$) reflects more adequately the structure of existing legal systems than its classical interpretation counterpart. The classical interpretation of $A \subseteq B$ says that any legally valid statement satisfying concept A also satisfies concept B, no matter how this concepts are related to the precedence relation. For example, in the three tiers involved in U.S. legal system (District court, Appellate court and Supreme court) the legally valid statements are strongly related by natural precedence order. If A is taken to be a concept meaning *Environmental* crimes and B means *Grave* crimes, then $A \subseteq B$ may not hold at all. A district court would not judge a situation of an oil-company that spoiled a beach in town as a grave crime. Note that at the federal level, the spoiling of natural resources is a grave crime. When considering the intuitionistic interpretation the subsumption $A \subseteq B$ holds, for any valid legal statement superceding "oil-company spoiled the coast and is sentenced to provide a compensation to the county" is superceded by a federal valid legal statement classified as a grave crime. In the Supreme court, the oil-company would be sentenced for commiting a grave crime.

$\mathcal{ALC}$ [1] is the basic description logic $\mathcal{AL}$ (for Attributive Language) plus the notion of complement $\mathcal{C}$ (negation) of arbitrary concepts. $\mathcal{ALC}$ is a fragment of classical first-order logic with just unary and binary relation symbols. Since $\mathcal{ALC}$ can be embedded into L_2 with constants, $\mathcal{ALC}$ has the finite model property and, hence, its satisfiability problem is decidable in NTIME($2^{O(n)}$), an upper complexity bound [2]. The intuitionistic version of $\mathcal{ALC}$, $i\mathcal{ALC}$, was proposed in [9, 8] to deal with legal reasoning. It has also the same upper bound.

The decidability to the nonmonotonic reasoning is preserved by imposing an order [4, 5] over the application of default rules that respects the Exceptions-First Principle [13] keeping also into account the knowledge represented by the description logic $i\mathcal{ALC}$. This principle is a key to solve the anomalous extension problem as the one introduced by the Yale Shooting Problem [11].

$i\mathcal{ALC}$ is closed related to many known intuitionistic modal and hybrid logic,

as it is discussed in [10] and in [3]. However, $i\mathcal{ALC}$ alone does not take care of every aspect of legal reasoning. It takes care of the task of sentence justification when a set of laws is considered. However, the legal procedure involves more than that. In a lawsuit, there are steps that take care of validating which are the valid legal statements to be considered. These are quite complex tasks that the agents involved in the lawsuit have to help to. The agents act in a default based way, since the process of including valid legal statements require to verify the (possibly) validity of other legal statements before accepting a (new) valid legal statement. $\mathsf{DD}_{i\mathcal{ALC}}$ is motivated by the aim of providing a logical tool to be used with this purpose. As an immediate illustration of a default reasoning in this case is the *default judgement* of the U.S. Civil Law that states that either party that fails to take action by the other party is sentenced.

We conclude by summing up our results: the definition of decidable description default logics that are expressible enough to formalize practical reasoning in knowledge bases as the hierarchical ontologies with exceptions, specially the ones that deals with legal knowledge and reasoning. The order we use among defaults allows one to avoid anomalous extensions and incoherent theories. We also present the main algorithms to build their extensions with a complexity analysis.

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