An Intuitionisticaly based Description Logic

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What does it mean the term “Law”? Two main (distinct) approaches

- What does count as the “unit of law”? Open question, a.k.a. “The individuation problem”.
- (Raz 1972) What is to count as one “complete law”?
- Taking all (existing) legally valid statements as a whole. This totality is called “the law”, a perfect legal world.
- Legal Positivism tradition (Kelsen 1934). Question: Natural coherence versus Knowledge Management resulted coherence.
- Taking into account all individual legally valid statement as individual laws.
- Facilitates the analysis of structural relationship between laws, viz. Primary and Secondary Rules.
- The second seems to be quite adequate to Legal AI.

Why we do not consider Deontic Modal Logic?

- Deontic logic approach to legal knowledge representation brings us paradoxes (contrary-to-duty paradoxes)
- Norms should not have truth-value. An individual law is not a deontic statement, it is not even a proposition. (Kelsen, Alchourrón etc)

Basic Motivations

- Description Logic is among the best logical frameworks to represent knowledge.
- Powerful language expression and decidable.
- *ALC*, as a basic *DL*, might be considered to legal knowledge representation if it can deal with the paradoxes.
- Considering a jurisprudence basis, classical *ALC* it is not adequate to our approach.

Our approach

- An intuitionistic version of *ALC* tailored to represent legal knowledge.
- Dealing with the paradoxes.
- A proof-theoretical basis to legal reasoning and explanation.
- **laws** are inhabitants of a universe that must be formalized.
- Propositions are about laws and not the laws themselves.

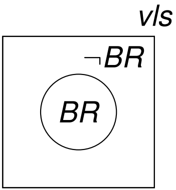
Haeusler, De Paiva, Rademaker (2010-2011).

Formalization of a Legal System following the second approach

- The first-class citizens of any Legal System are vls. Only vls inhabit the legal world.
- There can be concepts (collections of laws) on vls and relationships between vls. For example: PIL_{BR}, *CIVIL*, *FAMILY*, etc, can be concepts. *LexDomicilium* can be a relationship, a.k.a. a legal connection.
- The relationships between concepts facilitates the analysis of structural relationships between laws, viz. legal connections.
- A special relationships between laws is modeled as the natural precedence between laws, e.g. “ Peter is liable” precedes “Peter has a renting contract”.

Intuitionistic versus Classical logic

- Which version is more adequate to Law Formalization?
- The extension of an *ALC* concept is a **Set**.



Intuitionistic versus Classical logic (cont.)

Classical Negation: $\neg\phi \vee \phi$ is valid for any ϕ.

In *BR*, 18 is the legal age — *BR* contains all vls in Brazil

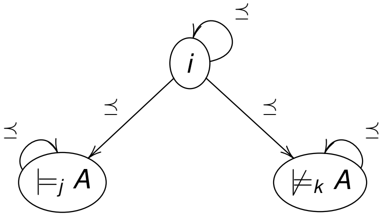
"Peter is 17"

"Peter is liable" $\notin$ *BR* iff "Peter is liable" $\in \neg$*BR*

Classical negation forces the "Peter is liable" be valid in some legal system outside Brazil.

Intuitionistic versus Classical logic (cont.)

The Intuitionistic Negation $\models_i \neg A$, iff, for all j, if $i \preceq j$ then $\not\models_j A$



$$\not\models_i \neg\neg A \to A \quad \text{and} \quad \not\models_i A \vee \neg A$$

Intuitionistic versus Classical logic (cont.)

- An Intuitionistically based approach to Law. Neither "Peter is liable"$\notin$ *BR* nor "Peter is liable"$\in \neg$*BR*.
- $pl \in \neg BR$ means $pl : \neg BR$ means $\mathcal{I}, pl \models \neg BR$ or $\forall z.\ z \succeq pl$ we have $z \not\models BR$.
- In other words, there is no z with $z \succeq pl$ such that $\mathcal{I}, z \models BR$. There is no vls in *BR* dominating "Peter is liable".

The logical framework for legal theories formalization

- Binary (Roles) and unary (Concepts) predicate symbols, $R(x, y)$ and $C(y)$.
- Essentially propositional (Tboxes), but may involve reasoning on individuals (Aboxes), expressed as "$i : C$".
- Semantics: Provided by a structure $\mathcal{I} = (\Delta^{\mathcal{I}}, \preceq^{\mathcal{I}}, \cdot^{\mathcal{I}})$ closed under refinement, i.e., $y \in A^{\mathcal{I}}$ and $x \preceq^{\mathcal{I}} y$ implies $x \in A^{\mathcal{I}}$. "$\neg$" and "$\sqsubseteq$" must be interpreted intuitionistically .
- It is not First-order Intuitionistic Logic. It is a genuine Hybrid logic.

Language

$$C, D ::= A \mid \bot \mid \top \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid C \sqsubseteq D \mid \exists R.C \mid \forall R.C$$

Below, A are general assertions and N nominal assertions for ABOX reasoning. Formulas (F) also includes subsumption of concepts interpreted as propositional statements.

$$N ::= x : C \mid x : N \qquad A ::= N \mid xRy \mid x \leq y \qquad F ::= A \mid C \sqsubseteq D$$

where x and y are nominals, R is a role symbol and C, D are concepts. In particular, this allows $x : (y : C)$, which is a perfectly valid nominal assertion with x begin its the outer nominal.

Interpretation

The interpretation $\mathcal{I}$ is lifted from atomic concepts to arbitrary concepts via:

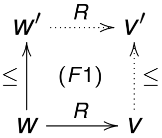
$$
\begin{array}{rl}
\top^{\mathcal{I}} & =_{df} \Delta^{\mathcal{I}} \\
\bot^{\mathcal{I}} & =_{df} \emptyset \\
(\neg C)^{\mathcal{I}} & =_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. x \preceq y \Rightarrow y \notin C^{\mathcal{I}}\} \\
(C \sqcap D)^{\mathcal{I}} & =_{df} C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} & =_{df} C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(C \sqsubseteq D)^{\mathcal{I}} & =_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. (x \preceq y \text{ and } y \in C^{\mathcal{I}}) \Rightarrow y \in D^{\mathcal{I}}\} \\
(\exists R.C)^{\mathcal{I}} & =_{df} \{x \mid \exists y \in \Delta^{\mathcal{I}}. (x, y) \in R^{\mathcal{I}} \text{ and } y \in C^{\mathcal{I}}\} \\
(\forall R.C)^{\mathcal{I}} & =_{df} \{x \mid \forall y \in \Delta^{\mathcal{I}}. x \preceq y \Rightarrow \forall z \in \Delta^{\mathcal{I}}. (y, z) \in R^{\mathcal{I}} \Rightarrow z \in C^{\mathcal{I}}\}
\end{array}
$$

Restrictions on the Interpretations

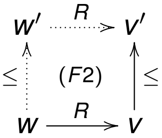
Following the semantics of IK, the structures $\mathcal{I}$ are models for $i\mathcal{ALC}$ if they satisfy two frame conditions:

- **F1** if $w \leq w'$ and wRv then $\exists v'.w'Rv'$ and $v \leq v'$
- **F2** if $v \leq v'$ and wRv then $\exists w'.w'Rv'$ and $w \leq w'$

The above conditions are diagrammatically expressed as:



and



Deductive Reasoning in *iALC*

$$\frac{}{\Delta, \delta \Rightarrow \delta} \qquad\qquad \frac{}{\Delta, x : \bot \Rightarrow \delta}$$

$$\frac{\Delta, xRy \Rightarrow y : \alpha}{\Delta \Rightarrow x : \forall R.\alpha}\ \forall\text{-r} \qquad\qquad \frac{\Delta, x : \forall R.\alpha, y : \alpha, xRy \Rightarrow \delta}{\Delta, x : \forall R.\alpha, xRy \Rightarrow \delta}\ \forall\text{-l}$$

$$\frac{\Delta \Rightarrow xRy \qquad \Delta \Rightarrow y : \alpha}{\Delta \Rightarrow x : \exists R.\alpha}\ \exists\text{-r} \qquad\qquad \frac{\Delta, xRy, y : \alpha \Rightarrow \delta}{\Delta, x : \exists R.\alpha \Rightarrow \delta}\ \exists\text{-l}$$

$$\frac{\Delta, \alpha \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqsubseteq \beta}\ \sqsubseteq\text{-r} \qquad\qquad \frac{\Delta_1 \Rightarrow \alpha \qquad \Delta_2, \beta \Rightarrow \delta}{\Delta_1, \Delta_2, \alpha \sqsubseteq \beta \Rightarrow \delta}\ \sqsubseteq\text{-l}$$

$$\frac{\Delta \Rightarrow \alpha \qquad \Delta \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqcap \beta}\ \sqcap\text{-r} \qquad\qquad \frac{\Delta, \alpha, \beta \Rightarrow \delta}{\Delta, \alpha \sqcap \beta \Rightarrow \delta}\ \sqcap\text{-l}$$

$$\frac{\Delta \Rightarrow \alpha}{\Delta \Rightarrow \alpha \sqcup \beta}\ \sqcup_1\text{-r} \qquad\qquad \frac{\Delta, \alpha \Rightarrow \delta \qquad \Delta, \beta \Rightarrow \delta}{\Delta, \alpha \sqcup \beta \Rightarrow \delta}\ \sqcup\text{-l}$$

$$\frac{\Delta, \alpha \Rightarrow \beta}{\forall R.\Delta, \exists R.\alpha \Rightarrow \exists R.\beta}\ \text{p-}\exists \qquad\qquad \frac{\Delta \Rightarrow \alpha}{\forall R.\Delta \Rightarrow \forall R.\alpha}\ \text{p-}\forall$$

$$\frac{\Delta \Rightarrow \delta}{x : \Delta \Rightarrow x : \delta}\ \text{p-N}$$

All propositional rules have their nominal version.

Using *iALC* to formalize Conflict of Laws in Space

Peter and Maria signed a renting contract. The subject of the contract is an apartment in Rio de Janeiro. The contract states that any dispute will go to court in Rio de Janeiro. Peter is 17 and Maria is 20. Peter lives in Edinburgh and Maria lives in Rio.

Only legally capable individuals have civil obligations:

$PeterLiable \preceq ContractHolds@RioCourt$, shortly, $pl \preceq cmp$
$MariaLiable \preceq ContractHolds@RioCourt$, shortly, $ml \preceq cmp$

Concepts, nominals and their relationships:

BR is the collection of Brazilian Valid Legal Statements
SC is the collection of Scottish Valid Legal Statements
PIL_{BR} is the collection of Private International Laws in Brazil
ABROAD is the collection of *VLS* outside Brazil
LexDomicilium is a legal connection: the pair $\langle pl, pl \rangle$ is in *LexDomicilium*

Non-Logical Axiom Sequents

The sets Δ, of concepts, and Ω, of *iALC* sequents representing the knowledge about the case.

$$\Delta = \boxed{\begin{array}{ccc} ml : \mathrm{BR} & pl : \mathrm{SC} & pl \preceq cmp \\ ml \preceq cmp & pl \ \mathrm{LexDom} \ pl & \end{array}}$$

$$\Omega = \boxed{\begin{array}{c} \mathrm{PIL_{BR}} \Rightarrow \mathrm{BR} \\ \mathrm{SC} \Rightarrow \mathrm{ABROAD} \\ \exists \mathrm{LexD_1.L_1} \ldots \sqcup \exists \mathrm{LexDom.ABROAD} \sqcup \ldots \exists \mathrm{LexD_k.L_k} \Rightarrow \mathrm{PIL_{BR}} \end{array}}$$

In Sequent Calculus

$$
\dfrac{
  \dfrac{
    \Delta \Rightarrow pl : \mathrm{SC}
    \qquad
    \dfrac{\Omega}{pl : \mathrm{SC} \Rightarrow pl : \mathrm{A}}
  }{
    \Delta \Rightarrow pl : \mathrm{A}
  }\,\mathit{cut}
  \qquad
  \Delta \Rightarrow pl \ \mathrm{LexD} \ pl
}{
  \Delta \Rightarrow pl : \exists \mathrm{LexD.A}
}\,\exists\text{-R}
\qquad
\dfrac{
  \dfrac{
    \dfrac{\exists \mathrm{LexD.A} \Rightarrow \exists \mathrm{LexD.A}}{\exists \mathrm{LexD.A} \Rightarrow \mathrm{PIL_{BR}}}\,\sqcup\text{-R}
    \qquad
    \dfrac{\Omega}{\mathrm{PIL_{BR}} \Rightarrow \mathrm{BR}}
  }{
    \exists \mathrm{LexD.A} \Rightarrow \mathrm{BR}
  }\,\text{cut}
}{
  pl : \exists \mathit{LexD.A} \Rightarrow pl : \mathit{BR}
}\,\text{p-N}
$$

$$
\dfrac{\Delta \Rightarrow pl : \exists \mathrm{LexD.A} \qquad pl : \exists \mathit{LexD.A} \Rightarrow pl : \mathit{BR}}{\Delta \Rightarrow pl : \mathrm{BR}}\,\text{cut}
$$

$$
\dfrac{
  \Delta \Rightarrow ml : \mathrm{BR}
  \qquad
  \dfrac{
    \dfrac{\Pi}{\Delta \Rightarrow pl : \mathrm{BR}}
    \qquad
    \dfrac{\Omega}{ml : \mathrm{BR}, pl : \mathrm{BR} \Rightarrow cmp : \mathrm{BR}}
  }{
    \Delta, ml : \mathrm{BR} \Rightarrow cmp : \mathrm{BR}
  }\,\mathit{cut}
}{
  \Delta \Rightarrow cmp : \mathrm{BR}
}\,\mathit{cut}
$$

Previous versions

- Consider the instance of $\sqsubseteq$-r rule

$$\frac{x : C \Rightarrow x : D}{\Rightarrow x : C \sqsubseteq D} \sqsubseteq\text{-r}$$

- with the semantics from (Hylo 2010):

$$\Theta, \Gamma \Rightarrow \gamma \equiv \Theta, \Gamma \models \gamma \equiv$$
$$\forall \mathcal{I}.((\forall x.\mathcal{I}, x \models \Theta) \implies \forall x.(\mathcal{I}, x \models \Gamma \implies \mathcal{I}, x \models \gamma))$$

  where Θ is the TBox. It is not sound.

- The conclusion states that for all $\mathcal{I}$ and worlds z with $x \preceq_{\mathcal{I}} z$ and $\mathcal{I}, z \models C$ we have $\mathcal{I}, z \models D$. The premise on the other hand only says that if for all $\mathcal{I}$ if $\mathcal{I}, x \models C$ we have $\mathcal{I}, x \models D$. This does not imply anything about the $\preceq$-successors of x.
- The counter model $\mathcal{I} = \{x, z\}$ with $x \preceq z$ and $x : \{C, D\}, z : \{C, \neg D\}$. So $C \sqsubseteq D$ not valid in x.

Solution: intuitionistic semantic of a sequent

- The semantics of the sequent $\Theta, \Gamma \Rightarrow \delta$ is $\Theta, \Gamma \models \delta$.
- We write $\Theta, \Gamma \models \delta$ if it is the case that:

$$\forall \mathcal{I}.((\forall x.\ \mathcal{I}, x \models \Theta) \Rightarrow \forall \vec{z} \succeq Nom(\Gamma, \delta).(\mathcal{I}, \vec{z} \models \Gamma \Rightarrow \mathcal{I}, \vec{z} \models \delta)$$

where $\vec{z}$ denotes a vector of variables $z_1, \ldots, z_k$ and $Nom(\Gamma, \delta)$ is the vector of all outer nominals occurring in each nominal assertion of $\Gamma \cup \{\delta\}$. x is the only outer nominal of a nominal assertion $\{x : \gamma\}$, while a (pure) concept γ has no outer nominal.

Soundness

Proposition

If $\Theta, \Gamma \Rightarrow \delta$ *is provable in* $SC_{\rangle\mathcal{ALC}}$ *then* $\Theta, \Gamma \models \gamma$.

Proof: We prove that each sequent rule preserves the validity of the sequent and that the initial sequents are valid.
Note: for each rule, we can derive the soundness of its non-nominal version from the proof of soundness of its nominal version.

Completeness

We shown the relative completeness of $SC_{i\mathcal{ALC}}$ regarding the axiomatic presentation (Plotkin 1986, Simpson 95,Fisher 1984):

0.	all substitution instances of theorems of IPL
1.	$\forall R.(C \sqsubseteq D) \sqsubseteq (\forall R.C \sqsubseteq \forall R.D)$
2.	$\exists R.(C \sqsubseteq D) \sqsubseteq (\exists R.C \sqsubseteq \exists R.D)$
3.	$\exists R.(C \sqcup D) \sqsubseteq (\exists R.C \sqcup \exists R.D)$
4.	$\exists R.\bot \sqsubseteq \bot$
5.	$(\exists R.C \sqsubseteq \forall R.C) \sqsubseteq \forall R.(C \sqsubseteq D)$
MP	If C and $C \sqsubseteq D$ are theorems, D is a theorem too.
Nec	If C is a theorem then $\forall R.C$ is a theorem too.

It is sufficient to derive in $SC_{i\mathcal{ALC}}$ the axioms 1–5. All instances of IPL theorems can also be proved using only propositional rules. The MP rule is a derived rule using the cut rule. The Nec rule is the p-$\forall$ rule in the system with Δ empty.

THANK YOU