An Intuitionsistally based Description Logic

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Classical Description Logic has been widely used as a basis for ontology creation and reasoning in many knowledge specific domains, including Legal AI. As in any other domain, consistency is an important issue for legal ontologies. However, due to its inherently normative feature, coherence (consistency) in legal ontologies is more subtle than in other domains. Consistency, or absence of logical contradictions, seems more difficult to maintain when more than one law system can judge a case. This is called a conflict of laws. There are some legal mechanisms to solve these conflicts, some of them stating privileged fori, other ruling jurisdiction, etc. In most of the cases, the conflict is solved by admitting a law hierarchy or a law precedence. Even using these mechanisms, coherence is still a major issue in legal systems since each layer in this legal hierarchy has to be consistent. As consistency is a direct consequence of how one deals with logical negation and subsumption. Negation and subsumption play a central role in ontology coherence.

An adequate intuitionistic semantics for negation in a legal domain comes to the fore when we take legally valid individual statements as the inhabitants of our legal ontology. This allows us to elegantly deal with particular situations of legal coherence, such as conflict of laws, as those solved by Private International Law analysis. In [7, 5, 8] we present an Intuitionistic Description Logic, called iALC for Intuitionistic ALC (for Attributive Language with Complements, the canonical classical description logic system). A labeled sequent calculus for iALC based on a labeled sequent calculus for ALC [10], was also presented. In these previous articles, we discussed the jurisprudence foundation of our system, and show how we can perform a coherence analysis of “Conflict of Laws in Space” by means of iALC. This conflict happens when several laws can be applied, with different outcomes, to a case depending on the place where the case occurs. Typical examples are those ruling the rights of a citizen abroad.

In [6], we presented the semantics of iALC on the framework for constructive modal logics presented by Simpson [12] and adapted to description languages by Paiva [4]. We apply this logic to the problem of formalizing legal knowledge.

Description Logics are an important knowledge representation formalism, unifying and giving a logical basis to the well known AI frame-based systems of the eighties. Description logics are very popular right now. Given the existent
and proposed applications of the Semantic Web, there has been a fair amount of
down into finding the most well-behaved system of description logic that has the
broadest application, for any specific domain. Description logics tend to come in
families of logical systems, depending on which concept constructors you allow
in the logic. Since description logics came into existence as fragments of first-
order logic chosen to find the best trade-off possible between expressiveness and
tractability of the fragment, several systems were discussed and in the taxonomy
of systems that emerged the $\mathcal{ALC}$ has come to be known as the canonical one.
The basic building blocks of description logics are concepts, roles and individuals.
Think of concepts as unary predicates in usual first-order logic and of roles as
binary predicates, used to modify the concepts.

As discussed in [4], considering versions of constructive description logics
makes sense, both from a theoretical and from a practical viewpoint. There
are several possible and sensible ways of defining constructive description logics,
whether your motivation is natural language semantics (as in [1]) or Legal AI
(as in [7]). As far as constructive description logics are concerned, Mendler
and Scheele have worked out a very compelling system $c\mathcal{ALC}$ [9], based on the
constructive modal logic $\mathcal{CK}$ [1], a favorit[2] system of ours. However in this
note we follow a different path and describe a constructive version of $\mathcal{ALC}$,
based on the framework for constructive modal logics developed by Simpson
(the system $\mathcal{IK}$) in his phd thesis [12] (For a proof-theoretic comparison between
the constructive modal logics $\mathcal{CK}$ and $\mathcal{IK}$ one can see [11]).

Our motivation, besides Simpson’s work, is the framework developed by
Brainer and de Paiva in [2] for constructive Hybrid Logics. We reason that
having already frameworks for constructive modal and constructive hybrid log-
ics in the labelled style of Simpson, we might end up with the best style of
constructive description logics, in terms of both solid foundations and ease of
implementation. Since submitting this paper we have been told about the mas-
ter thesis of Clément [3] which follows broadly similar lines. Clément proves
soundness and completeness of the system called $i\mathcal{ALC}$ and then provides a fo-
cused version of this system, a very interesting development, as focused systems
are, apparently, very useful for proof search.

Our Sequent Calculus for $i\mathcal{ALC}$ was first presented in [5] where we briefly
described the immediate properties of this system and most importantly we
discuss a case study of the use of $i\mathcal{ALC}$ in legal AI.

This article corrects and extends the presentation of $i\mathcal{ALC}$ appearing in all
previous articles. It points out the difference between $i\mathcal{ALC}$ and the intuitionis-
tic hybrid logic presented in [4]. Completeness and soundness proofs are revised.
A discussion on the computational complexity of $i\mathcal{ALC}$ is also taken.

References

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This system has categorical semantics, which are not very easy to obtain for modal logics.
REFERENCES


