An Intuitionisticaly based Description Logic

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Classical Description Logic has been widely used as a basis for ontology creation and reasoning in many knowledge specific domains, including Legal AI. As in any other domain, consistency is an important issue for legal ontologies. However, due to its inherently normative feature, coherence (consistency) in legal ontologies is more subtle than in other domains. Consistency, or absence of logical contradictions, seems more difficult to maintain when more than one *law system* can judge a case. This is called a *conflict of laws*. There are some legal mechanisms to solve these conflicts, some of them stating privileged fori, other ruling jurisdiction, etc. In most of the cases, the conflict is solved by admiting a law hierarchy or a law precedence. Even using these mechanisms, coherence is still a major issue in legal systems since each layer in this legal hierarchy has to be consistent. As consistency is a direct consequence of how one deals with logical negation and subsumption. Negation and subsumption play a central role in ontology coherence.

An adequate intuitionistic semantics for negation in a legal domain comes to the fore when we take legally valid individual statements as the inhabitants of our legal ontology. This allows us to elegantly deal with particular situations of legal coherence, such as conflict of laws, as those solved by Private International Law analysis. In [7, 5, 8] we present an Intuitionistic Description Logic, called $i\mathcal{ALC}$ for Intuitionistic $\mathcal{ALC}$ (for Attributive Language with Complements, the canonical classical description logic system). A labeled sequent calculus for $i\mathcal{ALC}$ based on a labeled sequent calculus for $\mathcal{ALC}$ [10], was also presented. In these previous articles, we discussed the jurisprudence foundation of our system, and show how we can perform a coherence analysis of "Conflict of Laws in Space" by means of $i\mathcal{ALC}$. This conflict happens when several laws can be applied, with different outcomes, to a case depending on the place where the case occurs. Typical examples are those ruling the rights of a citizen abroad.

In [6], we presented the semantics of $i\mathcal{ALC}$ on the framework for constructive modal logics presented by Simpson [12] and adapted to description languages by Paiva [4]. We apply this logic to the problem of formalizing legal knowledge.

Description Logics are an important knowledge representation formalism, unifying and giving a logical basis to the well known AI frame-based systems of the eighties. Description logics are very popular right now. Given the existent

and proposed applications of the Semantic Web, there has been a fair amount of work into finding the most well-behaved system of description logic that has the broadest application, for any specific domain. Description logics tend to come in families of logical systems, depending on which concept constructors you allow in the logic. Since description logics came into existence as fragments of first-order logic chosen to find the best trade-off possible between expressiveness and tractability of the fragment, several systems were discussed and in the taxonomy of systems that emerged the $\mathcal{ALC}$ has come to be known as the canonical one. The basic building blocks of description logics are *concepts*, *roles* and *individuals*. Think of concepts as unary predicates in usual first-order logic and of roles as binary predicates, used to modify the concepts.

As discussed in [4], considering versions of *constructive* description logics makes sense, both from a theoretical and from a practical viewpoint. There are several possible and sensible ways of defining *constructive* description logics, whether your motivation is natural language semantics (as in [4]) or Legal AI (as in [7]). As far as *constructive* description logics are concerned, Mendler and Scheele have worked out a very compelling system $c\mathcal{ALC}$ [9], based on the constructive modal logic CK [1]), a favorite[1] system of ours. However in this note we follow a different path and describe a constructive version of $\mathcal{ALC}$, based on the framework for constructive modal logics developed by Simpson (the system IK) in his phd thesis [12] (For a proof-theoretic comparison between the constructive modal logics CK and IK one can see [11]).

Our motivation, besides Simpson's work, is the framework developed by Braüner and de Paiva in [2] for constructive Hybrid Logics. We reason that having already frameworks for constructive modal and constructive hybrid logics in the labelled style of Simpson, we might end up with the best style of constructive description logics, in terms of both solid foundations and ease of implementation. Since submitting this paper we have been told about the master thesis of Clément [3] which follows broadly similar lines. Clément proves soundness and completeness of the system called $i\mathcal{ALC}$ and then provides a focused version of this system, a very interesting development, as focused systems are, apparently, very useful for proof search.

Our Sequent Calculus for $i\mathcal{ALC}$ was first presented in [5] where we briefly described the immediate properties of this system and most importantly we discuss a case study of the use of $i\mathcal{ALC}$ in legal AI.

This article corrects and extends the presentation of $i\mathcal{ALC}$ appearing in all previous articles. It points out the difference between $i\mathcal{ALC}$ and the intuitionistic hybrid logic presented in [4]. Completeness and soundness proofs are revised. A discussion on the computational complexity of $i\mathcal{ALC}$ is also taken.

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